Displacement Current

Frederick David Tombe,
Belfast, Northern Ireland, United Kingdom,
Formerly a Physics Teacher at
College of Technology Belfast, and
Royal Belfast Academical Institution,
sirius184@hotmail.com
7th October 2008, Belfast
(6th December 2008 Amendment, Davao City, Mindanao, Philippines)

Abstract. Displacement current is the term in Maxwell's modified version of Ampère's Circuital Law that enables the electromagnetic wave equation to be derived. It was originally conceived by Maxwell in connection with displacement of the electric particles in his sea of molecular vortices. It was conceived to exist in deepest space and not necessarily to be confined to the immediate vicinity of an electric current circuit.

Nowadays, displacement current is introduced as being the term that is needed to make Ampère's Circuital Law consistent with conservation of charge, and it is deemed not to be a real current. Maxwell on the other hand had already added displacement current to Ampère's Circuital Law prior to considering any such matters.

It will now be shown that the modern approach to displacement current is heavily flawed and that displacement current makes no difference whatsoever to the issue of the applicability of Ampère's Circuital Law in charge varying situations.

Maxwell's Approach

I. Maxwell conceived the idea of displacement current in connection with elasticity. He had proposed a sea of molecular vortices to explain electromagnetic phenomena, and those vortices were surrounded by electric particles that acted as idle wheels. His views on displacement current can be read in the introduction to part III of his 1861 paper 'On Physical Lines of Force' (beginning at page 39 in the pdf file) at, http://vacuum-physics.com/Maxwell/maxwell_oplf.pdf

Maxwell was never satisfied that his molecular vortex model represented a totally accurate picture, and so his attempt to explain the detailed physical significance of displacement current in relation to the rotational aspect of his molecular vortices was somewhat vague. He seemed to be saying that the force involved in displacement current is a tangential force which alters the state of angular momentum of the vortices, and that electromagnetic radiation is therefore a propagation of fine-grained angular acceleration. The angular momentum **H** would therefore be at right angles and in phase with the tangential force **E**.

The 1937 Encyclopaedia Britannica article on 'Ether' discusses its structure in relation to the cause of the speed of light. It says, "*POSSIBLE STRUCTURE.__ The question arises as to what that velocity can be due to. The most probable surmise or guess at present is that the ether is a perfectly incompressible continuous fluid, in a state of fine-grained vortex motion, circulating with that same enormous speed. For it has been partly, though as yet incompletely, shown that such a vortex fluid would transmit waves of the same general nature as light waves _i.e., periodic disturbances across the line of propagation_ and would transmit them at a rate of the order of magnitude as the vortex or circulation speed* - - - - - - -"

Maxwell added displacement current to Ampère's Circuital Law in order to make it applicable to 'Total Current', but it is clear that he did not intend the applicability of this modified version of Ampère's Circuital Law to be restricted to the vicinity of electric current circuits. His follow up work indicates that he intended it to apply anywhere where electromagnetic radiation exists. There seems to be a popular idea circulating around that Maxwell conceived of displacement current in conjunction with the electric capacitor circuit, but this idea is not found in his original papers.

The Modern Textbook Approach

II. The modern textbook approach to displacement current is quite different to Maxwell's approach. It is based on the idea that Ampère's Circuital Law needs to be modified in order to comply with situations, such as that which arises in the capacitor circuit, in which charge density is varying with time. Displacement current is then added to one side of

Ampère's Circuital Law as an additional term, but it is added on the basis that it is not a real current.

The fact that modern displacement current is not a real current means that the Ampère's Circuital Law equation has been unbalanced by virtue of adding a new term to one side only. Ampère's Circuital Law takes the general form,

curl $\mathbf{B} = \mu\mathbf{J}$ **(Ampère's Circuital Law)** **(1)**

With displacement current added, it then takes the form,

curl $\mathbf{B} = \mu\mathbf{J} + \mu\varepsilon\partial\mathbf{E}/\partial t$ **(Ampère's Circuital Law amended)** **(2)**

The justification that is given by the textbooks for adding the extra term is, that in doing so, we will maintain consistency when we take the divergence of both sides of equation **(2)**. As a result of both the equation of continuity of charge,

div $\mathbf{J} = -\partial\rho/\partial t$ **(Equation of Continuity of Charge)** **(3)**

and of Gauss's Law,

div $\mathbf{E} = \rho/\varepsilon$ **(Gauss's Law)** **(4)**

it follows that the divergence of the two terms on the right hand side of equation **(2)** will sum to zero, as should indeed be the result of taking the divergence of a curl. It is an established mathematical theorem that the divergence of a curl is always zero.

This approach however creates two problems. First of all, the justification for unbalancing the equation is based on the philosophy that the end justifies the means. That is a highly dubious approach when it comes to interfering with equations that have already been derived in the state that they are in. A closer look at the situation further shows that the additional term does not address the issue which it is said to be addressing.

A time variation in the charge density ρ would have no effect on the value of the curl in equation **(1)**, because a curl only involves partial spatial derivatives. Partial spatial derivatives mean that we are freezing the situation in time. So the means don't even create the end that is being claimed. The textbooks are unbalancing an equation and then addressing the issue of charge conservation by a devious mathematical conjuring trick which looks good, but which ignores the fundamental fact that Ampère's Circuital Law is only designed to operate on the basis of constant charge density in the first place. When Maxwell derived Ampère's Circuital Law hydrodynamically at equation (9) in part I of his 1861 paper 'On Physical Lines of Force', he didn't even involve charge. There can be no justification whatsoever for adding an additional $\partial\mathbf{E}/\partial t$ term to one side of equation **(1)**. Any attempt to introduce a term of the form $\partial\mathbf{E}/\partial t$ to Ampère's Circuital Law will have to be based on extracting it from within the already existing **J** term.

The second problem is that if we accept displacement current based on the modern textbook logic, then the **E** term in displacement current will satisfy Gauss's Law,

$$\text{div}\,\mathbf{E} = \rho/\varepsilon \qquad \textbf{(Gauss's Law)} \qquad \textbf{(4)}$$

Yet for the purposes of using displacement current to derive the electromagnetic wave equation, we don't want to have div **E** being equal to ρ/ε. We want to have div **E** equal to zero, with **E** being equal to $-\partial\mathbf{A}/\partial t$. Ampère's Circuital Law is a Coriolis force equation. It is not a Gauss's law equation. Equation (5) in part I of Maxwell's 1861 paper demonstrates quite clearly that Gauss's law is quite distinct from the Coriolis force. The Modern textbooks are getting the fundamental forces mixed up.

The Polarization Approach

III. A current flows in a capacitor circuit. This in turn causes a linear polarization of the dielectric between the capacitor plates which blocks the current flow. Linear polarization is a self restoring elastic effect and it is roughly what Maxwell had in mind for displacement current. Maxwell considered displacement current to differ from free current in that the elasticity of the medium would cause the displacement current to grind to a halt. However, as regards electromagnetic radiation, the displacement in question would have to be an angular displacement as opposed to a linear displacement. And in that regard it is interesting to note that Maxwell's concept of polarization was not the straightforward linear effect that we have in mind.

In part III of Maxwell's 1861 paper, he says *"I conceived the rotating matter to be the substance of certain cells, divided from each other by cell-walls composed of particles which are very small compared with the* *cells, and that it is by the motions of these particles, and their tangential action on the substance in the cells, that the rotation is communicated from one cell to another."*

Clearly Maxwell was driving at magnetization even though the same introduction clearly talks about dielectric polarization. Maxwell seemed to be somewhat confused about the distinction between the two concepts. If we replace his molecular vortices with electron-positron dipoles in which the electrons and positrons in each dipole are in a state of mutual circular orbit, then we can see that a tangential action will cause angular acceleration which constitutes magnetization, whereas it would be a radial action that would be needed to cause linear polarization. See 'The Double Helix Theory of the Magnetic Field' at,

http://www.wbabin.net/science/tombe.pdf

Maxwell hadn't got the difference between these two kinds of disturbance clear in his mind, largely because he didn't have a clearly focused picture of his molecular vortex cells. There is a stretch disturbance (linear polarization) and a spin disturbance (magnetization). The displacement current that is associated with electromagnetic radiation has to be a magnetization effect and not a linear polarization effect.

Conclusion

IV. The modern day displacement current is a highly dubious virtual concept, and it bears no connection to what Maxwell had in mind.

Conservation of charge in a capacitor circuit is not an issue which is in anyway addressed by displacement current. Conservation of charge is a hydrodynamical issue that is catered for by Bernoulli's Principle whereby voltage and charge represent pressure and current represents velocity. Charge variation with time is not a matter which is catered for in any respect within the realm of Ampère's Circuital Law. If we wish to add a displacement current term to Ampère's Circuital Law then we must justify it in terms of real current just as Maxwell did.

And in doing so we must then conclude that electromagnetic radiation does not propagate in a vacuum but rather in a dense sea of rotating electron-positron dipoles.