Gravitation and the Gyroscopic Force
(A hydrodynamical theory of gravity that accounts for the gyroscopic force)

Frederick David Tombe,
Belfast, Northern Ireland, United Kingdom,
Formerly a Physics Teacher at
College of Technology Belfast, and
Royal Belfast Academical Institution,
sirius184@hotmail.com
19th June 2006, Belfast
(24th January 2008 Amendment, Philippine Islands)

Abstract. Experimental evidence suggests that when a torque is applied to a spinning gyroscope such that the torque axis is perpendicular to the spin axis, then an induced torque will be generated in the gyroscope that is mutually perpendicular to both the spin axis, and to the applied torque axis. This induced gyroscopic torque exhibits the three way mutually perpendicular characteristics of the motion of a charged particle in a magnetic field. Applied mathematics textbooks do not however recognize the existence of induced gyroscopic torque as a distinct fundamental force in its own right. Textbooks assume that when a spinning gyroscope appears to be defying gravity, that this can be fully explained without having to recognize the existence of any additional forces beyond downward Newtonian gravity and upward normal reaction of a surface. This article proposes a general theory of gravity based on hydrodynamical principles which introduces three additional components that are not catered for by Newton’s law of gravitation. These three components link gravity directly with electromagnetism as well as fully accounting for the induced gyroscopic force in terms of the Coriolis force.

Mining the Vacuum for Information

I. Newton’s law of gravitation is only concerned with the radial component of gravity. The reason for this can be traced to Kepler’s law of areal velocity. Kepler observed that as each planet orbits the sun, it traces out an equal area in an equal time. In mathematical language, this was translated into the fact that no tangential component of acceleration is
involved in gravity. It is nevertheless of particular interest that the mathematical expression for areal velocity should happen to bear such a close relationship to the terms for tangential acceleration in the general acceleration equation (1) below. There must be a deeper significance to Kepler’s law of areal velocity, which we will now investigate. The general acceleration equation is obtained by differentiating a position vector in polar coordinates twice with respect to time. It takes the form,

$$a = \left(\frac{d^2r}{dt^2} - \omega^2 r\right)_{\text{radial}} + \left(2v \times \omega + r\frac{d\omega}{dt}\right)_{\text{tangential}}$$  \hspace{1cm} (1)$$

and it is a highly revealing equation. In equation (1) the symbol $r$ denotes distance from the origin in polar coordinates, $t$ denotes time, $\omega$ denotes angular speed, and $v$ denotes linear velocity. It should be noted that equation (1) makes no distinction between whether motion is gravitationally generated, electrically generated, or magnetically generated.

This equation enables us to extract an enormous amount of information out of what is officially considered to be nothing at all. To begin with, equation (1) assumes the existence of an inertial frame of reference in which the background stars appear to be fixed. This implies the existence of some kind of aethereal medium with which motion is measured to be relative to. The very concepts of position, velocity, and acceleration, imply the existence of particles moving in that aethereal medium. Equation (1) further tells us about the nature of the forces which act between particles in the aether, in relation to position, velocity, angular velocity, and angular acceleration. This connectivity between the aether and particles, manifested in equation (1) suggests that particles and the aether are two parts of the same overall entity.

The mathematical formulation of Newton’s law of gravity also assumes the existence of an inertial frame of reference, and so we may infer that the aether, the general acceleration equation, and Newton’s law of gravity are interconnected and that together they constitute one single phenomenon. The radial field lines for gravitational force that emanate from particles suggest to us that gravity may be caused by a flow of the aether into particles, and that particles themselves are merely sinks in the aether. Let us postulate a general theory of gravity based on the general equation of acceleration at equation (1).

When equation (1) was derived, it was assumed that empty space is rigid. We will now assume that space is dynamical and replace angular velocity $\omega$ with the vorticity vector $H$ which is related to $\omega$ through the equation,
The Hydrodynamical Theory of Magnetism

II. In his 1861 paper ‘On Physical Lines of Force’ [1],


James Clerk-Maxwell modelled the magnetic field hydrodynamically. He began with what would appear to be Bernoulli’s sea of aether vortices and he considered the alignment of these vortices, and relationships between density, velocity and pressure. Yet surprisingly, his concluding equation (5) in part I of his paper, that listed all the components of magnetic force, was not much more revealing than equation (3) above. Let us examine the terms in equation (3) above in conjunction with equation (5) in Maxwell’s 1861 paper.

The radial (or irrotational) component of equation (3) contains two terms. The first term, which we will refer to as the Newtonian term, is the inverse square law effect. The Newtonian term is position dependent and it clearly corresponds to the first term on the right hand side of equation (5) of Maxwell’s 1861 paper. The first term in equation (5) is the force that gives rise to the axial tension along magnetic lines of force and Maxwell demonstrated that if this term is irrotational then it must obey the inverse square law. He derived this fact at equation (21), and stated that this had already been shown by Coulomb. Maxwell stated after equation (19), which appears to be Laplace’s equation “Now it may be shown that equation (19), if true within a given space, implies that the forces acting within that space are such as would result from a distribution of centres of force beyond that space, attracting or repelling inversely as the square of the distance.”
This leaves us to speculate that a radially convergent and irrotational fluid flow in three dimensional space in the context of spherical symmetry, is a sufficient fact in its own right to determine the existence of an inverse square law of force.

The second term in the radial (irrotational) component is the centrifugal force which both Maxwell and Bernoulli considered to be a very real effect.

The centrifugal force is velocity dependent and it depends for its existence on actual tangential motion through the aether. Any two particles that possess mutual tangential speed will automatically experience mutual radial repulsion, and that radial repulsion is centrifugal force.

Maxwell used the concept of centrifugal force to account for the repulsion pressure in the equatorial plane of his molecular vortices. The equation for this repulsion pressure involves the square of the circumferential velocity in his vortices, and this ‘velocity squared’ relationship carries through to the second term of the right hand side of equation (5) in his 1861 paper. This term is used to explain pure magnetic force on unmagnetized matter and it establishes an Archimedes’ principle of magnetism that would account for the difference between diamagnetic materials and ferromagnetic materials. This equation of magnetism has now been lost as it doesn’t appear today in any of the standard textbooks of electromagnetism. See “Archimedes’ Principle in the Electric Sea” at,


The two components of the radial term when combined together, lead us to a differential equation for which the solution is an ellipse, a parabola, or a hyperbola.

The first term in the tangential component is the Coriolis/gyroscopic force. See section VI of ‘The Coriolis Force in Maxwell’s Equations’ at,


The gyroscopic force is a velocity dependent force and it occurs when a moving particle actually moves through rotating aether. The Coriolis/gyroscopic force appears as parts (3) and (4) of equation (5) in Maxwell’s 1861 paper.
The second tangential component is determined by angular acceleration of the aether relative to a particle and it appears to be missing a counterpart in Maxwell’s equation (5). It would certainly not seem to appear that this angular acceleration term is accounted for by the fifth term of equation (5) in Maxwell’s 1861 paper. The fifth term in equation (5) is simply implying that an element will be urged in the direction in which the hydrostatic pressure diminishes.

So despite the fact that Maxwell arrived at equation (5) of his 1861 paper using a sea of solenoidally aligned aether whirlpools, it would appear that he made less conclusions in terms of the fundamental aethereal forces than if he had merely used calculus on the pure dynamical and compressible aether. Maxwell’s mathematics didn’t seem to tie in as neatly with his excellent physical model as he would have perhaps wished it to have done.

An enormous amount of information can be obtained about the aether from one single position vector. The aether plays the fundamental role in all the forces of nature. Maxwell was very much on the right tracks. Electromagnetism does indeed require a solenoidally aligned sea of aether vortices and Maxwell was able to see reasonably well how this sea of vortices distributed the fundamental aethereal forces. But he didn’t need to begin with a sea of vortices in order to establish the mathematics behind the fundamental aethereal forces. All he needed to do for that purpose was to consider the hydrodynamics of the pure aether itself.

Interestingly, Maxwell did realize by part II of his 1861 paper ‘On Physical Lines of Force’[1] that aether hydrodynamics is meaningless without involving particles. So in part II, Maxwell introduced electrical particles to surround his aethereal vortices. He realized that he needed to introduce electrical particles in order to account for the continued existence of his aethereal vortices, which would otherwise have no justification for their existence.

We now know that electromagnetism requires the agency of a sea of rotating electron-positron dipoles in which electrons constitute aether sinks, and positrons constitute aether sources.
The Hydrodynamical Theory of Gravity

III. Equation (58) in part II of Maxwell’s 1861 paper [1], establishes a force of the form,

\[ E = \frac{dA}{dt} \]  \hspace{1cm} (The Unified Field Theory) \hspace{1cm} (4)

where \( A \) is attributed the properties of momentum. By analogy with this equation, let us propose a hydrodynamical theory of gravity in which the aether imparts its acceleration to a particle. (See Appendix A regarding escape velocity)

We will choose the vector \( A \) to represent aether velocity in order to highlight the analogy with Maxwell’s equation (58) and we will use the vector \( v \) for the velocity of a particle moving in the aether. We will write the hydrodynamical equation of gravity as,

\[ g = \frac{dA}{dt} \]  \hspace{1cm} (The Hydrodynamical Equation of Gravity) \hspace{1cm} (5)

or in differential form using the fly-wheel/vorticity equation \( \text{curl} \ A = H \),

\[ \text{curl} \ g = \frac{dH}{dt} \]  \hspace{1cm} (The Law of Magnetic Induction) \hspace{1cm} (6)

Using theorems from vector calculus (see Appendix B) we can expand and obtain the equation,

\[ \frac{dA}{dt} = \frac{\partial A}{\partial t} - v \times H + \text{grad}(A \cdot v) \] \hspace{1cm} (7)

As in the case of equation (1) we commence with a single vector \( A \), differentiate it with respect to time, and we obtain an enormous amount of physical information. Equation (7) was derived using exactly the same aethereal medium as for equation (1), and so it follows that there can be no new physics involved, and as such we should be able to match the three terms on the right hand side of equation (7), with the four terms on the right hand side of equation (3).

The first term on the right had side of equation (7) is known as the local term, and it only involves partial time derivatives. Let us propose that the local term applies to situations in which the aether directly imparts its acceleration to a particle. We can see how the Newtonian term of the radial component, and the angular acceleration term of the tangential
component might be covered by the local term. In both these two cases, the acceleration involves a change of magnitude of speed in the direction of flow.

The second and third terms on the right hand side of equation (7) are known as the convective terms. Convective terms are velocity dependent terms in which we consider motion in relation to spatial changes in A, while time is frozen. They are both such that the velocity of the particle relative to the aether actually induces the acceleration. They might be better understood as forces that arise out of cutting across the aether flow. The first of these two terms is clearly the Coriolis/gyroscopic force, and hence we are left with no choice but to conclude that the third term, $\nabla(A \cdot v)$, on the right hand side of equation (7) must the centrifugal force.

We can now summarize equation (7) as,

$$\frac{dA}{dt} = \frac{\partial A}{\partial t_{(irrotational)}} + \frac{\partial A}{\partial t_{(rotational)}} - v \times H + \nabla(A \cdot v)$$

(8)

since the Newtonian term is irrotational, and the angular acceleration term is rotational. Hence, using $\psi$ to denote gravitational potential, we can write,

$$g = \nabla \psi + \frac{\partial A}{\partial t_{(angular \ acceleration)}} - v \times H + \nabla(A \cdot v)$$

(9)

and we can compare this with the electromagnetic Lorentz force equation which occurs at equation (77) in Maxwell’s 1861 paper and also at equation (D) of the original eight Maxwell’s equations in his 1865 paper [2],

$$E = -\nabla \psi - \frac{\partial A}{\partial t} + v \times H \quad \text{(Lorentz Force)}$$

(10)

The conclusion is that we have managed to mine as much information out of empty space as all the great masters of the nineteenth century together managed to find out on the subject of magnetic force. (Equation (9) of course still needs to be weighted for ‘mass to charge’ ratio and for magnetic permeability in order for it to be applicable within the sphere of electromagnetism. The magnetic permeability is effectively the areal density of the electron-positron dipoles and when multiplied by the vorticity $H$, this will lead to the magnetic flux density $B$ which then becomes a measure of the fine-grain angular momentum of the magnetic field).
As explained in section II, the $\text{grad}(A \cdot v)$ (centrifugal) term was acknowledged by Maxwell to account for magnetic force on all unmagnetized materials, but he didn’t include it in equation (D) of his original set of eight ‘Maxwell’s Equations’ [2]. The $\text{grad}(A \cdot v)$ term is completely absent in most electromagnetic textbooks, although it is introduced in applied mathematics textbooks when the Lagrangian method is being used to solve Lorentz force problems.

**Deficiencies in modern Gravitational and Magnetic theory**

IV. By comparing equation (9) and equation (10), we can summarize the current deficiencies in both gravitational theory and electromagnetic theory.

In gravitation, we are missing both the Coriolis/gyroscopic component and the angular acceleration component. The Coriolis/gyroscopic component is sadly missing from all contemporary accounts relating to the theory of gyroscopes.

Kepler’s law of areal velocity tells us that large scale vortex effects have been predominantly absorbed into the fine-grain electron-positron vortices of the magnetic field but that is no guarantee that total absorption occurs everywhere in the universe. Strange spiral effects are indeed observed in distant galaxies.

Kepler’s laws leave us in no doubt that centrifugal force force is a real force. The solution to a Keplerian elliptical orbit unequivocally requires the action of two distinct radial forces. It requires a radially inward inverse square law force and a radially outward centrifugal force. Yet modern physics is teaching us that centrifugal force is only a fictitious force. This is a very bad misunderstanding.

This misunderstanding results in the fact that in modern electromagnetism we are missing the centrifugal component. Maxwell himself believed that centrifugal force was real and it played a very important role in his vortex sea model. But unfortunately he didn’t include this effect mathematically in his famous eight summarizing equations [2].
It was mentioned in section II of this article, how Maxwell used the centrifugal effect to account for the force acting on unmagnetized materials. Maxwell also used the centrifugal effect when it came to giving his physical explanation for the magnetic force acting on a current carrying wire. He explained the force on a current carrying wire, in terms of a pressure from behind, arising out of a centrifugal force that occurs as a result of his molecular vortices trying to expand in the equatorial plane. Maxwell also used fine-grain centrifugal force to explain ferromagnetic and electromagnetic repulsion. He pointed out that when two magnets are repelling each other that the magnetic lines of force spread outwards and away from each other. Where these lines meet laterally, they will repel each other due to centrifugal force in the equatorial plane of the molecular vortices.

Ironically the opposite was the case as regards the Coriolis/gyroscopic force. Although Maxwell included the mathematical form in equation [D] of his original eight ‘Maxwell’s Equations’ of 1864 [2], he failed to explicitly identify it as the Coriolis force. Maxwell also unwittingly used the Coriolis force to derive Ampère’s Circuital Law at equation (9) in his 1861 paper [1]. It was explained in ‘The Double Helix Theory of the Magnetic Field’,

http://www.wbabin.net/science/tombe.pdf

how the Coriolis force plays an instrumental role in creating the solenoidal alignment of the electron-positron dipoles around an electric current.

A final irony is the fact that the mathematical expression for the Coriolis/gyroscopic force in electromagnetism was eventually credited to Lorentz, who came much later, and it is wrongly used nowadays, but to a good approximation, to explain the force on a current carrying wire. The Coriolis/gyroscopic force can ideally only be used to explain the force on a current carrying wire that doesn’t have its own magnetic field. Maxwell as we know more accurately used fine-grain centrifugal force for this purpose.

The Gyroscopic Force

V. The gyroscopic force occurs when a spinning gyroscope is forced to rotate about an axis that is perpendicular to its axis of spin. The applied
torque in this situation has the effect of rotating the circumferential motion of the spinning gyroscope at right angles, through the aether. A force is generated that is mathematically identical to the Coriolis force \( \mathbf{mv} \times \mathbf{\omega} \), with \( \mathbf{\omega} \) referring to the angular velocity induced by the gravitational torque and \( \mathbf{v} \) referring to the average circumferential velocity of the gyroscope, and this induces a torque at right angles to both the axis of spin and the applied torque. It is this induced gyroscopic force which results in precession and nutation when a gyroscope with one end fixed, is subjected to a downward Newtonian gravitational torque. This induced torque is a convective aspect of gravity and it has the effect of curling the Newtonian aspect of gravity and pushing the gyroscope first sideways, and then upwards, resulting in precession and nutation. This real version of the Coriolis force cannot be fictitiously simulated by viewing a spinning gyroscope from a rotating frame of reference.

There is no recognition of distinct gyroscopic forces in modern textbooks on classical mechanics. It is officially accepted in orthodox circles that the apparent gravity defying stunts of the gyroscope can be perfectly explained by classical Newtonian gravitational theory. Yet anybody holding a spinning gyroscope by the gimbals and attempting to rotate it about an axis perpendicular to the axis of spin will find that it swings at right angles to the applied torque, as if a new torque has somehow been induced. Most people who carry out this simple experiment realize that an additional unrecognized force has come into play. Indeed the very fact that a gyroscope actually rises against traditional gravity during nutation, is sufficient evidence to suggest that an untraditional gravity has been induced which has overridden Newtonian gravity. See this web link regarding Professor Eric Laithwaite’s claims that a gyroscope looses weight during a forced precession.


Professor Laithwaite believed that in gyroscopes there is an extra force involved that is similar in principle to the electromotive force induced by the cutting of magnetic field lines in the theory of electromagnetic induction. The general theory of gravity at equation (3) would indicate that Professor Laithwaite was thinking along the correct lines.

The standard method for solving the ‘one end fixed’ heavy symmetrical top problem is to use the Lagrangian method. A modern textbook will set out to explain precession and nutation purely in terms of Newtonian gravitation. It will acknowledge that the torque of Newtonian gravity is acting downwards, and it will acknowledge the fact that this downward
torque leaves certain aspects of the motion constant. But the analysis will assume precession and nutation to be taking place without explaining why. The motion will then be analyzed based on conservation of energy and the possible inter relationships of the Euler angles and their time derivatives. There is a complete lack of mention of any mechanism that could actually cause precession or nutation to occur, contrary to the downward Newtonian force. A modern textbook will never acknowledge the existence of the gravitational convective gyroscopic force. If the $A_v$ velocity dependent potential, which is used in the Lagrangian for the Lorentz force, were to be introduced into the Lagrangian for a gyroscope, this might improve the situation somewhat.

One should not expect that the full set of eight ‘Maxwell’s Equations’ that appeared in Maxwell’s 1864 paper ‘A Dynamical Theory of the Electromagnetic Field’ [2] should be applicable to the gyroscope. We should only expect equations (B), (D), and (G) to be applicable, because a gyroscope doesn’t involve any concept equivalent to electric displacement, and hence neither does it involve electric current. For magnetic purposes, the aether is filled with tiny vortices. These tiny vortices are not involved in gyroscopic theory. In gyroscopic theory the gyroscope that is undergoing applied torque, is effectively one large vortex. In magnetism the electric current acts like a driving belt wrapped around the dipole vortices. The equivalent to electric current in a gyroscope is the mechanism that brings about the applied torque.

**The Sleeping Top**

VI. The sleeping top is a particularly interesting example of how modern textbooks purport to explain precession and nutation using only Newtonian gravity. The textbooks begin by considering the situation of a spinning top erected vertically on a surface. It is accepted that there will be no resultant force. The Lagrangian method is then employed and they surprisingly manage to establish that below a certain angular velocity, the top will precess and nutate. Yet if no additional forces are brought into play, theoretically speaking, the top should continue standing upright even if it stops spinning altogether. The fact that the situation would be highly unstable is of no relevance whatsoever. The Lagrangian method only tells us that precession and nutation are commensurate with the fact that energy is conserved. It tells us absolutely nothing regarding how precession and nutation might be caused. The force that causes the precession is the $\mathbf{F} = \mathbf{v} \times \mathbf{H}$ force. The $\mathbf{F} = \mathbf{v} \times \mathbf{H}$ force acts on a particle in
such a manner as to change the particle’s direction, and hence it doesn’t
act such as to change the kinetic energy of the particle. The gyroscopic \( F = v \times H \) force is a direction changing force and it changes the direction of
a falling pivoted gyroscope such as to make it precess and nutate.

It is quite incredible that the precession of a pivoted gyroscope is taught
in universities in the absence of the very force that causes the precession.
The fact that the gyroscopic \( F = v \times H \) doesn’t interfere with the total
mechanical energy means that it is compatible with a Lagrangian analysis
purely from an energy conservation perspective. But leaving the
gyroscopic force out of the analysis removes the vital dimension of cause.

**Where ‘Like Charges’ Attract**

VII. In 1982 a unified theory of electrostatics and gravity was derived
that can be seen in section IV of this web link,


The principle behind the derivation was that inertial mass is a cumulative
quantity based on the amount of matter in a body and that it has a
retarding effect on all externally applied forces in which this force acts
differently on the positively charged particles in the body, than it does on
the negatively charged particles in the body. Gravitational mass was
referred to as gravitational charge, and due to the mono-polarity of
gravitational charge it was considered as a separate quantity and summed
beside electric charge in the numerator. This derivation therefore united
gravity and electricity mathematically but failed to account for any
physical linkage between the two theories.

The reason why gravitational charge and electric charge were treated as
separate entities was because of the fact that in Coulomb’s law, it is
believed that like charges repel each other, whereas in Newton’s law of
gravity, we know that like charges attract each other. This will now have
to be reviewed in the light of aether hydrodynamics. Within the context
of aether hydrodynamics, it is absolutely impossible for two sinks to
mutually repel each other. It is also impossible that a particle would
constitute an aether sink in one aether and simultaneously constitute an
aether source in another aether. Aether hydrodynamics unequivocally
tells us that gravity and electricity constitute one single theory. The fact
that two sinks must attract each other, whereas we also have laboratory
evidence that demonstrates to us that two negatively charged bodies repel each other, means that we have no choice but to investigate and discover what the intervening mechanism is that appears to reverse the mutual attraction. See ‘Gravity Reversal and Atomic Bonding’ at,


Like charges should only repel when they are positive (ie. when they constitute aether sources) and this means that all matter normally deemed to be neutral must actually be negatively charged in order to account for gravity.

There would appear to be at least some evidence that negative charges attract each other. There is the evidence of the existence of Cooper Pairs in superconductors. There is the evidence of the fact that neutral atoms can form attractive bonds with each other. There is the evidence that cathode rays don’t mutually repel.

Just as two sinks must mutually attract, it also follows that a sink and a source will either attract or repel each other depending on whether the sink or the source is dominant. This means that in a rotating electron positron dipole, the negative charge of the electron must be greater than the positive charge of the positron, hence causing the dipole to possess an excess negative charge, and therefore to constitute a gravitational monopole. The gravitational charge of the electric sea would then account for why it is gravitationally entrained to the Earth, as is manifestly evident simply by looking at a diagram of the Earth’s magnetosphere.

In the 1937 Encyclopaedia Britannica article on electricity it says regarding Benjamin Franklin (1706-1790) “- - - He supposed therefore that two vitreously electrified bodies would repel each other and that a vitreously electrified body would attract a resinously electrified body but he did not expect two resinously electrified bodies to repel each other - - - “.

In the same article, it says regarding Aepinus (1724-1802) “Aepinus also suggested that the attractive forces between two uncharged bodies might be very slightly greater than the repulsive forces and that this difference might be the cause of gravitation.”
Appendix A

The velocity field of the aether does not impose its velocity on a particle. The velocity of a particle is arbitrary. Prof. Reg Cahill [3] has argued that a Newtonian gravitational field must necessarily have an associated velocity field and he linked this velocity field to escape velocity. Ian Montgomery in Australia told me that we will just have to accept the mysterious fact that the aether imparts its acceleration but not its velocity to a particle. In doing so, we begin to see the rationale behind escape velocity in terms of a particle overcoming the aether flow.

The equation for escape velocity takes the form,

\[ V = \sqrt{\frac{2GM}{r}} \]  \hspace{1cm} (1A)

To see if equation (1A) is commensurate with the acceleration due to gravity, we must consider it in relation to the equation,

\[ g = \frac{V}{dV/dr} \] \hspace{1cm} (2A)

Since,

\[ \frac{dV}{dr} = \frac{1}{2\left(\sqrt{\frac{2GM}{r}}\right)} \times \left(-\frac{2GM}{r^2}\right) \] \hspace{1cm} (3A)

the product in (2A) will become,

\[ \frac{V}{dV/dr} = -\frac{GM}{r^2} \] \hspace{1cm} (4A)

which means that a fluid field based on escape velocity is commensurate with the inverse square law associated with irrotational radially convergent gravitational acceleration.

A mysterious aethereal fluid seems to be pouring in from elsewhere in the universe. The idea that gravity is a converging (sink) flow of aether has been suggested at least as early as 1931 by O.G. Hilgenbergbackin.
Appendix B

The gradient of a scalar product of two vectors is given by the standard vector identity,

\[ \text{grad}(\mathbf{A} \cdot \mathbf{v}) = \mathbf{AX} \text{curl} \mathbf{v} + \mathbf{vX} \text{curl} \mathbf{A} \]

\[ + (\mathbf{A} \cdot \text{grad})\mathbf{v} + (\mathbf{v} \cdot \text{grad})\mathbf{A} \]  
(1B)

Since \( \mathbf{v} \) represents arbitrary particle motion, the first and the third terms on the right hand side of equation (1B) will vanish, and from the relationship \( \text{curl} \mathbf{A} = \mathbf{H} \), we obtain,

\[ \text{grad}(\mathbf{A} \cdot \mathbf{v}) = \mathbf{vXH} + (\mathbf{v} \cdot \text{grad})\mathbf{A} \]  
(2B)

Hence,

\[ (\mathbf{v} \cdot \text{grad})\mathbf{A} = -\mathbf{vXH} + \text{grad}(\mathbf{A} \cdot \mathbf{v}) \]  
(3B)

Since,

\[ \frac{d\mathbf{A}}{dt} = \frac{\partial \mathbf{A}}{\partial t} + (\mathbf{v} \cdot \text{grad})\mathbf{A} \]  
(4B)

we obtain,

\[ \frac{d\mathbf{A}}{dt} = \frac{\partial \mathbf{A}}{\partial t} - \mathbf{vXH} + \text{grad}(\mathbf{A} \cdot \mathbf{v}) \]  
(5B)

showing that a single differentiation of a vector can yield all the aspects of magnetic force that were identified by the great masters of the 19th century.

References


http://www.zpenergy.com/downloads/Maxwell_1864_2.pdf


http://redshift.vif.com/JournalFiles/V11NO1PDF/V11N1CA1.pdf