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Aether Friction in the Planetary Orbits

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Abstract. **When a theory of electromagnetism promotes the idea that the medium for the propagation of light waves is an elastic solid comprised of electric particles, the question is always going to be asked as to why this medium would not generate friction in the planetary orbits, such as would cause the planets to spiral into the Sun. It would be impossible for a moving body to completely avoid any physical interaction with these electric particles, and so, in order to comply with Kepler's Laws of Planetary Motion, this interaction must be the actual cause of the inertial forces, as opposed to being the cause of any dissipative friction.**

The Inertial Path

I. The Barnett Effect, discovered in the year 1915 by American physicist Samuel Jackson Barnett, demonstrates that a magnetic field, albeit extremely weak, is generated around a spinning body, even if the body is electrically neutral **[1]**. It will now be investigated whether the inertial forces can be linked to this weak magnetic field. A body in motion will satisfy the equation,

$$m\ddot{\mathbf{r}} = m(\ddot{r} - r\dot{\theta}^2)\hat{\mathbf{r}} + m(2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\boldsymbol{\theta}} \qquad \textbf{(1)}$$

relative to any polar origin. See **Appendix I** for the derivation and interpretation of symbols. In the special case where $\ddot{\mathbf{r}} = 0$, a body will be undergoing its uniform straight-line inertial path and equation **(1)** then becomes Newton's first law of motion expressed in polar coordinates, relative to an inertial frame of reference. It can then be reduced to,

$$m\ddot{r}\hat{\mathbf{r}} = +mr\dot{\theta}^2\hat{\mathbf{r}} \qquad \textbf{(2)}$$

where $mr\dot{\theta}^2\hat{\mathbf{r}}$ is the ***centrifugal force***. Equation **(2)** can alternatively be written as,

$$m\ddot{r}\hat{\mathbf{r}} = +mh\hat{\mathbf{r}}/r^3 \qquad \textbf{(3)}$$

where h is a constant related to angular momentum. See **Appendix II** for the derivation. This means that a body in motion has a centrifugal force to every point in space, implying the existence of a centrifugal force field surrounding all moving bodies. Meanwhile, the transverse term $2m\dot{r}\dot{\theta}\hat{\boldsymbol{\theta}}$ is known as the ***Coriolis force***. The centrifugal force and the Coriolis force are known as ***inertial forces***.

It should be further noted that the centrifugal force field mentioned in the paragraph above must be solenoidal since the sum of all the centrifugal forces in the field is zero. This is similar in principle to what happens in the case of the magnetic field that surrounds a long straight current carrying wire. If we define the magnetic field intensity, **H**, with respect to a point on the wire and then add the values of **H** at every point in space, the resultant is zero.

In 1861, James Clerk Maxwell, in the preamble to Part I of his seminal paper entitled ***"On Physical Lines of Force"***, explained the magnetic field as a centrifugal force field **[2]**. He proposed that space is filled with tiny aethereal vortices that press against each other with centrifugal force while striving to dilate **[3]**, **[4]**, **[5]**, and he referred to this sea of vortices as the *luminiferous medium*. Maxwell further explained Ampère's Circuital Law as being the tendency of the rotation axes of these tiny vortices to form concentric solenoidal rings around an electric current, these rings of force being magnetic lines of force. Magnetic repulsion between like poles is then caused by the centrifugal force in the equatorial planes of the vortices at the interface between two adjacent magnetic fields, since this centrifugal force acts perpendicularly to the magnetic lines of force, which spread outwards between two like poles.

It is now suggested that these same tiny aethereal vortices are responsible for the inertial forces. As the luminiferous medium flows through the interstitial spaces between the atoms and molecules of all moving bodies, as like water flowing through a basket, this will generate a gyroscopic interaction akin to the principles lying behind Maxwell's explanation for Ampère's Circuital Law. This interaction leads to the formation of vortex rings, concentric on the line of motion, and centred on the moving body. This will be the weak magnetic field associated with the Barnett effect and it will exist in conjunction with a gyroscopic stability in the atoms and molecules of the moving body, resulting in the uniform straight-line inertial path **[6]**. In the case of two bodies in relative motion, unless they are bonded to the luminiferous medium in the vicinity of their centrifugal force fields, then they will have no direct physical impact on each other, but in the case of planetary bodies undergoing orbital motion, their gravitational fields should be strong enough to entrain the luminiferous medium, hence transferring the physical interaction to the interface between the two gravitational fields.

The Inertial Forces in the Unbonded State

II. When two terrestrial bodies do a fly-by past each other while undergoing their uniform straight-line inertial paths, they will possess a mutual centrifugal acceleration which is maximum at the point of closest approach. The magnitude of this mutual centrifugal acceleration as they continue along their straight-line inertial paths can be substantial, and it produces a physical effect that can be commandeered for useful purposes in mechanical devices. However, unless their gravitational fields are strong enough to entrain the luminiferous medium, then despite any centrifugal repulsion at the interface between the two centrifugal fields, the two bodies will not have any direct influence on each other, and so we must ascribe any centrifugal force in such circumstances to the direct gyroscopic interaction that the atoms and molecules of each of these two bodies have with the dense background sea of tiny aethereal vortices. This is an example of centrifugal force in the ***unbonded state***.

Where a force seeks to change only the speed of a body, but not its direction, the inertial mass of the body serves as a reactance to this force and the situation comes under the jurisdiction of Newton's second law of motion. No inertial forces will be involved. However, where an applied force seeks to change the direction of a body that is undergoing its uniform straight-line inertial path in the unbonded state, the situation will be different. When it comes to trying to curve the path of a moving body, this will be opposed by an inertial force, and this inertial force will depend, not only on the body's inertial mass, but also upon its velocity.

Inertial forces are measured relative to a point origin, and we normally choose the point origin that is significant to the applied force that the inertial force is opposing. For example, if a centripetal force or a central force is applied, we will normally use the origin of those forces. If a transverse force is applied, as for example when we roll a marble along a radial groove in a turntable that is undergoing a forced rotation, we will locate the origin at the centre of rotation. The transverse force impressed on the marble by the forced rotation will then be opposed by an inertial Coriolis force, to that origin.

In many cases, it is the inertial motion itself which pro-actively causes the inertial force, such as when a weight is being swung around in a circle on the end of a string. In this case, the centripetal force does not come into existence until the string is taut, and it is the centrifugal force which causes the string to become taut in the first place. It can therefore never be said that a centrifugal force is always a reaction to a centripetal force. It is often the action. We could accurately say that an inertial force is a force which is *at the ready for action* when a body is undergoing its uniform straight line inertial path. The centrifugal force field is already there, squeezing inwards on the motion from all sides. It acts perpendicularly to the direction of motion but can be resolved into any direction, reducing to zero along the line of motion. There is therefore no pressure fore and aft. Along the line of motion, a body will encounter no resistance from the dense background sea of aethereal vortices. Resistance from the sea only occurs when attempts are made to curve the path of motion.

Likewise, when a marble is rolled along a radial groove towards the centre of a freely rotating turntable, the Coriolis force is pro-active and it will cause the turntable to angularly accelerate, and hence to spin faster. The inertial forces exist in their own right, and they are a product of both the mass of a body and its velocity. Centrifugal force is actually the radial gradient of rotational kinetic energy, otherwise known as centrifugal potential energy. Its magnitude is independent of the magnitude of any external centripetal force that the body might be subjected to. The applied centripetal force will have its own magnitude already set, and it will determine a point origin. But the magnitude of the centrifugal force which opposes it from this origin will be determined by the mass and velocity of the moving body. Only in the special case where the centripetal force causes circular motion will the two opposing forces have the same magnitude. It is a common error to believe that a centrifugal force is the reaction in an equal and opposite action-reaction pair with a centripetal force. Even Newton wrongly believed this **[7]**, but Kepler's laws of planetary motion reveal that centrifugal force is totally independent of gravity **[8]**, **[9]**. We can see in a centrifuge machine that centrifugal force acts to fling the heavier particles to the rim where no centripetal force is even involved. So long as a body is forced to co-rotate in a rotating system, a centrifugal force will fling it radially outwards, and that is a physical reality.

Centrifugal Force between Electric Particles

III. Consider two electron-positron dipoles sitting side by side. In each dipole, the electron and the positron are orbiting each other in a circular orbit. They are each orbiting in the same plane and in the same direction, and they each have the same angular velocity. See ***"The Positronium Orbit in the Electron-Positron Sea"*** **[10]**. When the electron of one dipole passes the positron of the other dipole in the opposite direction, then, at the moment of closest approach, the electrostatic field lines should be connecting directly between the two particles, and so according to Coulomb's law there should be a force of attraction acting between them, as in the case of any two particles of opposite charge. If however the electrostatic force field, $\mathbf{E}_S$, is based on tension in an aethereal electric fluid that is flowing from the positron to the electron, their mutual transverse speed will induce a curl into the velocity field. Above a certain threshold of angular speed, the flow lines connecting the two dipoles will be cut, and the two separate regions of electric fluid, assumed to be inviscid, will now be shearing past each other in opposite directions. The dipoles will be striving to dilate and so the pressure emanating sideways from the opposing flow lines will push the two dipoles apart, since the aether cannot pass laterally through itself. A centrifugal repulsion will have taken the place of the electrostatic attraction. This centrifugal repulsion is akin to that with which the water in Newton's rotating bucket pushes outwards against its inside walls. This is the underlying principle behind both magnetic repulsion and electrostatic repulsion at the deepest fundamental level. Maxwell's tiny aethereal vortices will therefore now be identified with rotating electron-positron dipoles, **[11]**.

Centrifugal Force in Planetary Orbits

IV. In the case of the large planetary bodies where the gravitational fields are significant enough to entrain a pocket of the background sea of tiny electron-positron dipoles with them along their orbital paths, it would seem that the centrifugal force acting between two planets is physically induced at the interface of their respective gravitational fields by a shear interaction involving the centrifugal pressure arising in the electron-positron vortices that are present at this location. It will be a kind of hovercraft effect involving fine-grained gyroscopy, **[12]**, **[13]**. The mutual alignment in their equatorial planes, of the vortices at the interface, is attributed to the same principles that underly Maxwell's explanation for Ampère's Circuital Law. The tendency for two adjacent vortices, undergoing mutual motion, to align themselves in their equatorial planes, is a fundamental gyroscopic effect.

Hence, in cases where the gravitational fields of two planets share a common interface, the mutual centrifugal acceleration between them represents an actual physical repulsion, whereas in the case of two planets, such as Jupiter and Mars, whose gravitational fields do not encounter each other, the mutual centrifugal acceleration between them will be as per in the unbonded state described in section **II** above.

Friction in Space

V. The important thing is that the shear interaction between a moving body and Maxwell's background sea of vortices, or that between two gravitationally entrained regions of this sea of vortices, should account for the inertial forces rather than causing dissipative friction that would result in the planets spiralling into the Sun. It is clear that a body moving through a dense sea of electric particles is going to interact with those particles, and that energy will be transferred to the surroundings. It is necessary therefore to identify a possible circular energy flow mechanism. As a precedent, we are aware of the case of an inductance coil, and that when we switch off the electric power, the energy stored in the magnetic field flows back into the conducting wire again giving the electric current a final forward surge. Furthermore, any analogy between the magnetic forces and the inertial forces would be persuasive as to the possibility that inertial motion involves a shear interaction with the luminiferous medium similar to that which is observed when a charged particle moves through a magnetic field. In his 1861 paper ***"On Physical Lines of Force"***, Maxwell derived the lossless electromotive force,

$$\mathbf{E_L} = \mu\mathbf{v} \times \mathbf{H} \qquad \textbf{(4)}$$

where **H** is the magnetic intensity, which in Maxwell's theory corresponds to the vorticity of the tiny vortices. Meanwhile, the magnetic permeability, μ, is a quantity related to density, while **H** is equal to $2\boldsymbol{\omega}$, where $\boldsymbol{\omega}$ is angular velocity, which in this case should be that of the electric particles circulating around the edge of the vortices. Maxwell derived equation **(4)** in terms of differential centrifugal pressure in his sea of aethereal vortices. Meanwhile, the centrifugal force term in equation **(2)** in section **I** above can alternatively be written in the form,

$$\mathbf{F} = m\mathbf{v_t} \times \boldsymbol{\omega} \qquad \textbf{(5)}$$

where $\mathbf{v_t}$ is the transverse speed. If we cannot connect the inertial forces to the medium that is the cause of electromagnetic phenomena, then this will create serious problems for the planets in their orbits. The fact that equations **(4)** and **(5)** both describe a force that is velocity dependent, and which acts at right-angles to the causative motion, suggests that they are both induced by the same kind of lossless elasticity, albeit in different contexts. Ampère's Circuital Law describing how a magnetic field exists around an electric current would therefore appear to apply to inertial motion in just the same way.

The Coriolis Force

VI. While centrifugal force is a radial aspect of the uniform straight-line inertial path, the Coriolis force is a transverse aspect. In equation **(1)** in section **I**, the transverse component contains a Coriolis force in the form $2m\dot{r}\dot{\theta}\hat{\boldsymbol{\theta}}$ alternatively written as $2m\mathbf{v_r} \times \boldsymbol{\omega}$ where $\mathbf{v_r}$ is the radial velocity. In the unbonded state, it would seem that the Coriolis force is exclusively a consequence of the gyroscopic stability implicit in the uniform straight-line inertial path. It is a matter of interest also, that the Coriolis force is always cancelled by an equal and opposite inertial force, this being the underlying basis for the law of conservation of angular momentum. In planetary orbits, conservation of angular momentum is observed in the form of Kepler's second law, which is the law of equal areas being traced out in equal times.

Just as the centrifugal force manifests itself when a centripetal force steers a particle off its straight-line path, a Coriolis force likewise manifests itself when a transverse force steers a particle off its inertial path. This manifestation is best observed in the behaviour of gyroscopes. Although centripetal force due to the inter-molecular bonds within the gyroscope has already taken the situation outside of the uniform straight-line inertial path scenario, the action of the Coriolis force can still be observed when we subject the spinning gyroscope to a forced precession. In the case of a pivoted spinning gyroscope, gravity causes a forced precession which induces a Coriolis torque, and hence a precession at right-angles to the forced precession. Since the normal reaction of the pivot cancels one side of this Coriolis couple, the Coriolis force on the other side can make the centre of gravity rise upwards against gravity. The weight is therefore transferred from the centre of gravity to the pivot, making it easier on the wrist muscles when lifting a heavy spinning gyroscope. It is commonly accepted however that the gyroscope is not any lighter as such, but the induced Coriolis torque reduces the required momentary force needed to hold the gyroscope at the end of its stem.

An analysis of this situation was carried out in ***"Magnetic Repulsion and the Gyroscopic Force"***, **[14]**, and this article gives a further insight into the nature of the physical reaction that occurs when the luminiferous medium passes through the interstitial spaces between the atoms and molecules of ponderable matter, as like water passing through the holes in a basket. Gyroscopic stability in ponderable matter may involve a mechanism at a deeper molecular level on the nano-scale, akin to the aerodynamic P-factor (asymmetric blade effect) or the Magnus effect. So, while centrifugal force is the anti-gravity force in planetary orbits, the Coriolis force is the anti-gravity force in pivoted gyroscopes.

Meanwhile, just as centrifugal force manifests itself when a body is co-rotating in a rotating system, a Coriolis force likewise manifests itself when a constrained radial motion occurs in a rotating system. This situation is observed in atmospheric cyclones. The rotation of the Earth introduces angular momentum into the cyclone and the Coriolis force ensures conservation of angular momentum, manifested by a transverse deflection of a radial air current. In the case of artificial satellite orbits undergoing a non-circular orbit, the Coriolis force will simply be a geometrical fact of the inertial path since it is believed that the luminiferous medium passes right through the interstitial spaces between the atoms and molecules of the satellite. However, in the case of planets that entrain a large pocket of the luminiferous medium within their gravitational fields, and where the centrifugal force between planets is due to a fine-grained gyroscopic interaction involving the tiny rotating electron-positron dipoles at the interface between two gravitational fields, the Coriolis force will be absorbed into the general interaction. The Coriolis force will amount to equal and opposite transverse centrifugal forces pressing from each side. This will have the illusion that a Coriolis force has deflected the effects of gravity into the transverse direction in a process not dissimilar to how the Coriolis force deflects the effects of gravity sideways in a pivoted gyroscope [14]. This illusion may in fact be a reality in the deeper vortex interactions occurring among the rotating electron-positron dipoles at the interface between two gravitational fields.

We are looking at an extremely subtle fine-grained gyroscopic mechanism occurring in an elastic medium which serves as the medium for the propagation of light, while simultaneously, without contradiction, underlying Kepler's laws of planetary motion. We are looking at a medium which is a solid for the purposes of electromagnetic wave propagation, but which exhibits fluid characteristics in accommodating the planetary motions.

Conclusion

VII. In the classical limit, ignoring Einstein's ***"General Theory of Relativity"***, [15], the elasticity in space that is involved in electromagnetism and electromagnetic waves must conform to Newton's laws of motion and to Kepler's laws of planetary motion, otherwise the planets would spiral into the Sun. The inertial forces are closely related to the convective aspect of electromagnetic induction where, $\mathbf{E_L} = \mu\mathbf{v} \times \mathbf{H}$, while Newton's second law of motion is closely related to time-varying electromagnetic induction where, $\mathbf{E_K} = -\partial\mathbf{A}/\partial t$, with only the latter being involved in electromagnetic waves. It is now suggested that inertial motion is closely related to the same physical cause that underlies Ampère's Circuital Law.

According to James Clerk Maxwell, Ampère's Circuital law is about how a moving element interacts with the surrounding luminiferous medium in such a way as to induce vortex rings that amount to magnetic lines of force [2]. These lines of force form concentric solenoidal rings around the direction of motion. The manner of the physical interaction that induces these magnetic lines of force is dictated by hydrodynamics in the context of a sea of tiny aethereal vortices that are pressing against each other with centrifugal force while striving to dilate. It is now suggested that this same mechanism determines the uniform straight-line inertial path by a process involving gyroscopic stability, out of which forms a surrounding centrifugal force field. This is in fact a weak magnetic field, similar in principle to that discovered by S.J. Barnett in 1915, [1], in connection with a spinning neutral body. When a strong gravitational field entrains the sea of aethereal vortices, hence bonding it to the gravitating object such as an orbiting planet, the centrifugal force field then begins at the edge of the entrained zone. Hence at the interface between two orbiting planets, there will be a cushion of centrifugal repulsion which undermines the gravitational force of attraction.

The important thing however is, that if space is densely filled with electric particles in order to accommodate the propagation of light waves, then these electric particles must interact with matter in motion in such a way as to generate the inertial forces in line with Kepler's laws of planetary motion, rather than to be the cause of dissipative friction that would result in the planets spiralling into the Sun. This can be achieved if space is densely packed with tiny aethereal vortices, where each vortex comprises a single electron in orbit with a positron, hence forming a dense neutral dielectric sea.

References

[1] Barnett, S.J., ***"Magnetization by Rotation"*** Physical Review, Volume 6, Issue 4, page 239 (1915)

[2] Clerk-Maxwell, J., ***"On Physical Lines of Force"***, Philosophical Magazine, Volume XXI, Fourth Series, London, (1861)
http://vacuum-physics.com/Maxwell/maxwell_oplf.pdf
Equation (77) in this paper is Maxwell's electromotive force equation and it exhibits a strong correspondence to equation **(1)** above. The centrifugal and Coriolis terms in equation **(1)** correspond to the compound centrifugal force term $\mu\mathbf{v} \times \mathbf{H}$, while the other transverse term corresponds to $-\partial\mathbf{A}/\partial t$.

[3] Whittaker, E.T., ***"A History of the Theories of Aether and Electricity"***, Chapter 4, Pages 100-102, (1910)
"All space, according to the younger Bernoulli, is permeated by ***a fluid aether, containing an immense number of excessively small whirlpools.*** *The elasticity which the aether appears to possess, and in virtue of which it is able to transmit vibrations, is really due to the presence of these whirlpools;* ***for, owing to centrifugal force, each whirlpool is continually striving to dilate, and so presses against the neighbouring whirlpools****."*

[4] Lodge, Sir Oliver, ***"Ether (in physics)"***, Encyclopaedia Britannica, Fourteenth Edition, Volume 8, Pages 751-755, (1937)
http://gsjournal.net/Science-Journals/Historical%20PapersMechanics%20/%20Electrodynamics/Download/4105
In relation to the speed of light, *"The most probable surmise or guess at present is that* ***the ether is a perfectly incompressible continuous fluid, in a state of fine-grained vortex motion,*** *circulating with that same enormous speed. For it has been partly, though as yet incompletely, shown that such a vortex fluid would transmit waves of the same general nature as light waves— i.e., periodic disturbances across the line of propagation—and would transmit them at a rate of the same order of magnitude as the vortex or circulation speed"*

[5] O'Neill, John J., ***"PRODIGAL GENIUS, Biography of Nikola Tesla"***, Long Island, New York, 15th July 1944, quoting Tesla from his 1907 paper ***"Man's Greatest Achievement"*** which was published in 1930 in the Milwaukee Sentinel,
"Long ago he (mankind) recognized that all perceptible matter comes from a primary substance, of a tenuity beyond conception and filling all space - the Akasha or luminiferous ether - which is acted upon by the life-giving Prana or creative force, calling into existence, in never ending cycles, all things and phenomena. The primary substance, thrown into infinitesimal whirls of prodigious velocity, becomes gross matter; the force subsiding, the motion ceases and matter disappears, reverting to the primary substance".
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https://www.researchgate.net/publication/283225757_Magnetic_Repulsion_and_the_Gyroscopic_Force

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Appendix I

Consider a particle moving in an inertial frame of reference. We write the position vector of this particle relative to any arbitrarily chosen polar origin as,

$$\mathbf{r} = r\hat{\mathbf{r}} \qquad \textbf{(1A)}$$

where the unit vector $\hat{\mathbf{r}}$ is in the radial direction, and where r is the radial distance. Taking the time derivative and using the product rule, we obtain the particle's velocity,

$$\dot{\mathbf{r}} = \dot{r}\hat{\mathbf{r}} + r\dot{\theta}\hat{\boldsymbol{\theta}} \qquad \textbf{(2A)}$$

where $\hat{\boldsymbol{\theta}}$ is the unit vector in the transverse direction, and where $\dot{\theta}$ is the angular speed about the polar origin. Taking the time derivative again we obtain the expression for the particle's acceleration in the inertial frame,

$$\ddot{\mathbf{r}} = \ddot{r}\hat{\mathbf{r}} + \dot{r}\dot{\theta}\hat{\boldsymbol{\theta}} + \dot{r}\dot{\theta}\hat{\boldsymbol{\theta}} + r\ddot{\theta}\hat{\boldsymbol{\theta}} - r\dot{\theta}^2\hat{\mathbf{r}} \qquad \textbf{(3A)}$$

which can be rearranged as,

$$\ddot{\mathbf{r}} = (\ddot{r} - r\dot{\theta}^2)\hat{\mathbf{r}} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\boldsymbol{\theta}} \qquad \textbf{(4A)}$$

Appendix II

Kepler's second law is the law of areal velocity, which is essentially a statement that angular momentum is conserved, and it means that only radial forces are involved in planetary orbits. This can be proven in reverse as follows. If the total transverse acceleration is zero, then from equation **(4A)**,

$$2\dot{r}\dot{\theta} + r\ddot{\theta} = 0 \qquad \textbf{(5A)}$$

hence, multiplying across by r,

$$2r\dot{r}\dot{\theta} + r^2\ddot{\theta} = 0 \qquad \textbf{(6A)}$$

Since,

$$2r\dot{r}\dot{\theta} + r^2\ddot{\theta} = d/dt(r^2\dot{\theta}) = 0 \qquad \textbf{(7A)}$$

then,

$$r^2\dot{\theta} = L \qquad \textbf{(8A)}$$

where L is a constant which is twice the areal velocity, and so the areal velocity must be constant. Hence,

$$\dot{\theta} = L/r^2 \qquad \textbf{(9A)}$$

Substituting $\dot{\theta}$ in **(9A)** into **(4A)** in conjunction with **(5A)** we obtain,

$$\ddot{\mathbf{r}} = (\ddot{r} - L^2/r^3)\hat{\mathbf{r}} \qquad \textbf{(10A)}$$

which expresses the centrifugal force in terms an inverse cube law in radial distance from the polar origin and a constant, L^2, that is related to angular momentum. In the case of the uniform straight-line inertial path, where we have $\ddot{\mathbf{r}} = 0$, equation **(10A)** becomes,

$$\ddot{r}\hat{\mathbf{r}} = +h\hat{\mathbf{r}}/r^3 \qquad \textbf{(11A)}$$

where $h = L^2$ is a constant related to angular momentum. This means that relative to any arbitrarily chosen point in space, a particle undergoing uniform straight-line motion experiences a centrifugal acceleration to this point, dependent on the angular momentum relative to it, and this acceleration diminishes as per the inverse cube of the distance from the point.