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Faraday's Law of Electromagnetic Induction

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Abstract. **It will be shown how the magnetic vector potential, A, is a momentum which is central to Faraday's law of electromagnetic induction, and how the convective electromagnetic force E = v×B is the factor which enables the total time derivative to be used in Faraday's law.**

Introduction

I. If we start with a vector field, **A**, and take a total time derivative, this can be split into a partial time derivative and a convective term as per,

$$d\mathbf{A}/dt = \partial\mathbf{A}/\partial t + (\mathbf{v}.\nabla)\mathbf{A} \qquad \mathbf{(1)}$$

This is a standard theorem in applied mathematics. Now additionally there exists a vector identity formula for the gradient of the scalar product of two vectors as follows,

$$\nabla(\mathbf{A}.\mathbf{v}) = \mathbf{A}\times(\nabla\times\mathbf{v}) + \mathbf{v}\times(\nabla\times\mathbf{A}) + (\mathbf{A}.\nabla)\mathbf{v} + (\mathbf{v}.\nabla)\mathbf{A} \qquad \mathbf{(2)}$$

If, however, **v**, is an arbitrary particle velocity and not a vector field, then both the curl and the gradient of **v** will be zero. Equation **(2)** therefore reduces to,

$$\nabla(\mathbf{A}.\mathbf{v}) = \mathbf{v}\times(\nabla\times\mathbf{A}) + (\mathbf{v}.\nabla)\mathbf{A} \qquad \mathbf{(3)}$$

Substituting **(3)** into **(1)** we get,

$$d\mathbf{A}/dt = \partial\mathbf{A}/\partial t - \mathbf{v}\times(\nabla\times\mathbf{A}) + \nabla(\mathbf{A}.\mathbf{v}) \qquad \mathbf{(4)}$$

If we now define a vector, **E**, such that $\mathbf{E} = -d\mathbf{A}/dt$, and a vector, **B**, such that $\mathbf{B} = \nabla\times\mathbf{A}$, this leads to,

$$\mathbf{E} = -\partial\mathbf{A}/\partial t + \mathbf{v}\times\mathbf{B} - \nabla(\mathbf{A}.\mathbf{v}) \qquad \mathbf{(5)}$$

Taking the curl of **E**, and since the curl of a gradient is always zero, we get,

$$\nabla\times\mathbf{E} = -\partial\mathbf{B}/\partial t - (\mathbf{v}.\nabla)\mathbf{B} \qquad \textbf{(6)}$$

(see **Appendix I**)

and hence from equation **(1)**,

$$\nabla\times\mathbf{E} = -d\mathbf{B}/dt \qquad \textbf{(7)}$$

If **E** is a force, then this is the full Faraday's law involving both the time varying and convective aspects, and **A** must be a circulating momentum with **B** being the corresponding axial vector field. **A** is therefore the foundation stone of electromagnetism. Maxwell referred to it as the ***electromagnetic momentum*** and he identified it with Faraday's electrotonic state. It is referred to in modern textbooks as the ***magnetic vector potential***. Space must therefore be filled with tiny circulations with their mutual rotation axes tracing out the prevailing magnetic lines of force. The vector **A** can be shown to correspond to Maxwell's displacement current [1].

Conclusion

II. Faraday's law can be derived directly from hydrodynamics without any experimental input. However, in the case of the substantive equation **(5)** for the electromagnetically induced **E** field, we have a term $\nabla(\mathbf{A}.\mathbf{v})$ which we don't want if we wish to replicate that which appears in the textbooks. We got rid of it above by taking the curl of equation **(5)**, but in order to get rid of it in substance, we need to establish an appropriate physical context.

First of all, we will consider the vector **A** to be the momentum at any point in space of a fundamental electric fluid which pervades all of space. As regards equation **(1)**, we will conclude that the partial time derivative term $\partial\mathbf{A}/\partial t$ represents the acceleration of this primary aethereal fluid at any particular point, and that it imparts its acceleration to sinks and sources within it at that point. These sinks and sources constitute negative and positive particles. Particles do not however necessarily take on the velocity of the primary fluid. When particles move relative to the primary fluid, their acceleration relative to the fluid is given by the convective term $(\mathbf{v}.\nabla)\mathbf{A}$ in equation **(1)**. In the special case when $\nabla(\mathbf{A}.\mathbf{v})$ is equal to zero, $(\mathbf{v}.\nabla)\mathbf{A}$ then becomes equal to $\mathbf{v}\times\mathbf{B}$. Although the scalar product $\mathbf{A}.\mathbf{v}$ is used in Lagrangian mechanics as the potential function for $-\partial\mathbf{A}/\partial t + \mathbf{v}\times\mathbf{B}$, it makes no difference that the product is zero. It doesn't mean that either **A** or **v** has to be zero.

We need to establish a physical context in which $\mathbf{A}.\mathbf{v}$ is zero and the obvious case is where a particle moves perpendicularly to **A**, and where **A** has a non-zero curl such as would be the case if a particle were cutting across a

curved arc of flow. For such a situation to be sustained on a constant basis we would require space to be densely packed with tiny aethereal vortices, being the equivalent of a single rotation on the large scale [2], [3], [4]. It is proposed that this is what the cross-section of a magnetic field looks like and that $\mathbf{E} = \mathbf{v}\times\mathbf{B}$ is a deflection due to rotation that is closely related to the Coriolis force. Since curl $\mathbf{A} = \mu\mathbf{H} = \mathbf{B}$, the magnetic field intensity $\mathbf{H}$ is a vorticity, equivalent to $2\boldsymbol{\omega}$ in the context, where $\boldsymbol{\omega}$ is the angular velocity. The magnetic permeability, μ, is a weighting for the magnetic flux density $\mathbf{B}$, and so it represents the cross-sectional density of the magnetic field. Hence $\mathbf{E} = 2\mu\mathbf{v}\times\boldsymbol{\omega}$, which looks very like the familiar Coriolis force $\mathbf{F} = 2m\mathbf{v}\times\boldsymbol{\omega}$. The speed $\mathbf{v}$ is of course measured relative to the sea of aether vortices that pervades all of space.

Appendix I

It's a standard vector identity that the curl of a vector cross product expands as follows,

$$\nabla\times(\mathbf{v}\times\mathbf{B}) = \mathbf{v}(\nabla.\mathbf{B}) - \mathbf{B}(\nabla.\mathbf{v}) + (\mathbf{B}.\nabla)\mathbf{v} - (\mathbf{v}.\nabla)\mathbf{B} \qquad \mathbf{(1B)}$$

If $\mathbf{v}$ represents an arbitrary particle velocity, and since $\nabla.\mathbf{B} = 0$, this reduces to,

$$\nabla\times(\mathbf{v}\times\mathbf{B}) = -(\mathbf{v}.\nabla)\mathbf{B} \qquad \mathbf{(2B)}$$

References

[1] Tombe, F.D., ***"Displacement Current and the Electrotonic State"*** *(*2008) http://gsjournal.net/Science-Journals/Research%20PapersMechanics%20/%20Electrodynamics/Download/228

[2] Clerk-Maxwell, J., ***"On Physical Lines of Force"***, Philosophical Magazine, Volume XXI, Fourth Series, London, (1861) http://vacuum-physics.com/Maxwell/maxwell_oplf.pdf

[3] Whittaker, E.T., ***"A History of the Theories of Aether and Electricity"***, Chapter 4, pages 100-102, (1910)
"All space, according to the younger Bernoulli, is permeated by a fluid aether, containing an immense number of excessively small whirlpools. The elasticity which the aether appears to possess, and in virtue of which it is able to transmit vibrations, is really due to the presence of these whirlpools; for, owing to centrifugal force, each whirlpool is continually striving to dilate, and so presses against the neighbouring whirlpools."

[4] O'Neill, John J., ***"PRODIGAL GENIUS, Biography of Nikola Tesla"***, Long Island, New York, 15th July 1944, quoting Tesla from his 1907 paper ***"Man's Greatest Achievement"*** which was published in 1930 in the Milwaukee Sentinel,
"Long ago he (mankind) recognized that all perceptible matter comes from a primary substance, of a tenuity beyond conception and filling all space - the Akasha or luminiferous ether - which is acted upon by the life-giving Prana or creative force, calling into existence, in never ending cycles, all things and phenomena. The primary substance, thrown into infinitesimal whirls of prodigious velocity, becomes gross matter; the force subsiding, the motion ceases and matter disappears, reverting to the primary substance".
http://www.rastko.rs/istorija/tesla/oniell-tesla.html
http://www.ascension-research.org/tesla.html