Magnetic Repulsion and the Gyroscopic Force

(Manifestations of Centrifugal Force)

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Abstract. The counterintuitive gravity defying behaviour that is exhibited by a pivoted gyroscope suggests the involvement of an active spin-induced force, similar in nature to the magnetic force, $F = qv \times B$, and which cannot be predicted by Newtonian mechanics. The phenomenon of gyroscopic stability exhibits a strong reactance which cannot be accounted for by the moment of inertia. The physical connection between the inertial forces and magnetic repulsion will be investigated.

The Spinning Top

I. When an electrically charged particle falls vertically under gravity in a horizontal magnetic field, it will be deflected horizontally at right angles to the magnetic field lines. At its lowest point it will be moving horizontally at a speed which, at the same height, it would have been moving downwards if there had been no magnetic field. The magnetic force has diverted the effect of gravity sideways in much the same manner that a concave inclined plane does. The deflecting force, rather than merely superimposing upon the gravitational force, has had the effect of actually deflecting the entire gravitational force sideways.

On reaching its lowest point, the particle then begins to rise again. As it rises it loses speed and the radius of curvature of its path decreases. At maximum height the particle loops around in a retrograde direction and the cycle repeats over again, but after each cycle the particle will have advanced in the horizontal direction. The particle will trace out an average horizontal path containing a series of loops. If however we block the horizontal advance, the particle will fall to the ground like a stone since the magnetic force that would have prevented it from falling will be disengaged.

A spinning top is a gyroscope which is placed on a surface so that the combination of gravity acting downwards and the upward normal reaction cause a torque to act. Intuition tells us that this torque should pull the gyroscope right down, but if the angular speed is fast enough, that's not what happens. It follows from Newtonian mechanics that when the gyroscope begins to fall downwards, its angular momentum vector will move horizontally. That does not however automatically mean that the spin axis should move sideways to chase

the angular momentum vector. Whether it does or not and why, is the issue that lies at the centre of this investigation.

If we physically block the spinning top from moving sideways, then just like in the case of when we blocked the charged particle in the magnetic field from moving sideways, it will fall like a stone, and it doesn't matter that its angular momentum vector has been displaced sideways from the spin axis. However, if we do not prevent the top from moving sideways, the spin axis will counter intuitively deflect sideways to chase the angular momentum vector, and the top will defy gravity while the spin axis traces out a pattern of loops similar to that in the case of the charged particle falling through the magnetic field. This suggests that there is an active spin-induced force at play, similar in nature to the magnetic force $\mathbf{F} = \mathbf{q}\mathbf{v}\times\mathbf{B}$.

The Coriolis Force

II. The Coriolis force has a mathematical structure that is similar to that of the magnetic force $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$, especially bearing in mind that Maxwell linked the magnetic force intensity to angular velocity. [1], [2] Despite the conventional wisdom that a Coriolis force is merely an artefact of making observations from a rotating frame of reference, it will now be proposed that the Coriolis force is a real transverse force which acts when a radial motion occurs in conjunction with an angular velocity in a radial field. The radial velocity and the angular velocity must be independent of each other, yet physically connected. A common example occurs where a person sits on a rotating stool with outstretched arms. When they retract their arms, the inward radial motion of their arms is independent of, but physically connected to the rotatory motion, and so a Coriolis force is induced which causes the stool to spin faster.

Another example of a radial motion that is constrained to rotate occurs in the case of a precessing gyroscope. If we view a spinning gyroscope from the side, into its equatorial plane, and then apply a precessional torque along an axis that is pointing towards us, the rim velocity will be radial relative to the axis of the applied torque. It will be centrifugally directed on one side of the torque axis and centripetally directed on the other side. A Coriolis force will hence be induced in the same direction on either side of the torque axis. In conjunction with an equal and opposite Coriolis force on the far side of the gyroscope, a new torque will be induced which will be at right angles to the applied torque. This induced torque has the power to deflect the effects of gravity sideways.

Both of the examples described above involve real physical effects which can be observed from an inertial frame of reference and which can be analyzed in terms of an active Coriolis force. While the rotating stool scenario can also be analysed in terms of conservation of angular momentum, this is not so in the case of the gyroscope where the Coriolis force actually causes the angular momentum to change its direction.

The Coriolis force is very obviously an absolute physical effect and not, as is commonly taught, merely an artefact arising from making observations in a rotating frame of reference. It is a real force, every bit as real as its cousin the magnetic force. Introducing a rotating frame of reference into the analysis is like introducing a hall of mirrors which merely serves to confuse the whole issue. Nevertheless, although there are some exceptions [3], [4], it is unusual to see the Coriolis force being treated as a real force in the literature.

The Magnet and the Gyroscope

III. We'll consider a closed loop of electric current to be both a magnet and a gyroscope. We find however, that it doesn't possess the gravity defying ability that is possessed by the ordinary gyroscope. Neither the spin associated with the electric circulation nor the associated magnetic field can stop such an electromagnetic gyroscope from falling to the ground. A corollary of this is that two ordinary spinning gyroscopes sitting side by side with their rotation axes parallel don't repel each other as do two magnets with parallel magnetic axes.

These two facts combined point to the spin of the individual molecules within the ordinary gyroscope as being the source of the gyroscopic Coriolis force, whereas the source of the magnetic force must lie in the magnetic field in the space beyond atomic and molecular matter. Repulsion between two magnets must arise due to an effect which occurs at the interface between their two respective magnetic fields. Maxwell identified magnetic repulsion with centrifugal force in a sea of molecular vortices that serves as the medium for the propagation of light (the luminiferous medium). [1]

Centrifugal Force

IV. When two bodies possess a mutual transverse speed, there will be an inertial centrifugal force acting on each of the bodies relative to their common centre of mass. The centrifugal forces acting on each body will be equal and opposite, while the body with the larger mass will have the smaller centrifugal acceleration. Contrary to what is stated in the mainstream literature, inertial centrifugal force is a real force that obeys Newton's laws. If the two bodies are connected by a string, the string will be pulled taut, hence inducing a centripetal force pair on the two bodies.

Inertial centrifugal force does not form part of an action-reaction pair with centripetal force, and in situations where a centripetal force pair is induced in a constraint, it is only in the special case where circular motion ensues that it is equal in magnitude to the inertial centrifugal force pair. This should not however be confused with the fact that the centrifugal force that actually acts on the constraint itself is equal and opposite to the induced centripetal force. The centrifugal force that acts on the constraint is not the same thing as the inertial centrifugal force that acts on the moving bodies themselves, although they both form part of a single process with the inertial force being the primary cause.

The Rattleback

V. The physical reality of the centrifugal force can be demonstrated with a device known as a rattleback. A ratteback is a boat shaped object that sits on a hard surface and the only officially recognized forces acting on it are gravity, normal reaction, and friction. The crucial feature of a rattleback is that it is asymmetrically shaped in relation to rocking it on a horizontal axis, and it is important that the hard surface provides static friction so as to enable a see-saw rocking motion to occur. Let's now assume that centrifugal force is a real force. When the rattleback rocks, an equal and opposite pair of inertial centrifugal forces act on either side of the rock axis. The asymmetrical shape ensures that these centrifugal forces act out of the plane of the rocking motion. On any given side of the rock axis, the centrifugal force always acts in the same direction, whether during the upward part of the motion or during the downward part of the motion. The result is that the centrifugal force pair produces a torque, hence causing the rattleback to rotate in the horizontal plane in a direction determined by the chirality of the asymmetry.

When a rattleback rotates in a horizontal plane, there will always be a certain amount of rocking as well. The centrifugally induced torque will then act to precess the combined angular momentum of the two superimposed modes of rotation. If we rotate the rattleback in its preferred direction, the centrifugal torque will stabilize the rotation, but if we rotate it contrary to its preferred direction of rotation, the centrifugal torque will act to precess the combined angular momentum until the rotation has completely reversed its direction. This is observed initially by an increase in the rocking in conjunction with a decrease in the initial rotation. At the moment when the rocking amplitude reaches its maximum, the rotation will change directions. It will then speed up in the new direction as the rocking fades away. This counter intuitive display would not be possible if centrifugal force were not a real force.

Some rattlebacks work two ways. No matter which way we rotate it, it will ultimately reverse its direction. This is because there is an additional asymmetry

with respect to rotation in the horizontal plane. When this situation occurs, the precession caused by the centrifugal torque never finds a symmetrical axis, and so the cycle repeats indefinitely until it is damped out by sliding friction.

The Underlying Cause

VI. We have attributed the gravity defying ability of a gyroscope to the Coriolis force and the spin reversal tendency in the rattleback to the centrifugal force, but we have so far not identified the underlying physical cause of these inertial forces. The commonality that underlies both of these inertial forces, and also the magnetic repulsive force, is the compound centrifugal force principle rooted in Maxwell's sea of molecular vortices. The sea of tiny molecular vortices, which constitutes the luminiferous medium, fills all of space including the interstitial regions between the atoms and molecules of ponderable matter. The luminiferous medium should not be confused with the pure aether which is the actual stuff of which the molecular vortices themselves are comprised. The luminiferous medium is a sea of tiny aether vortices. [1], [5], [6], [7]

The fundamental principle is that these vortices press against each other with centrifugal force while striving to dilate. The magnetic force and the inertial forces arise when motion causes an asymmetry in the fine-grained centrifugal force that is being exerted on ponderable matter by this background sea of tiny aethereal vortices. When such asymmetry arises, the fine-grained centrifugal force presses differentially on each side, resulting in a large scale 'compound centrifugal force', alternatively known as a 'Coriolis force'.

In the case of the magnetic force, the skeleton principle can be illustrated by considering two molecular vortices sitting side by side and rotating in the same direction. The centrifugal force that they exert on each other is due to the tendency to expand, which is in turn related to the mutual circumferential speed of the electric particles that circulate around the edge of the vortices. If another particle moves between these two vortices, due to its mutual transverse speed with respect to the particles circulating around the edge of the vortices, it will experience a centrifugal force. The mutual transverse speed will be greater on one side than on the other, and so the particle will deflect at right angles to its direction of motion. In a magnetic field the tiny vortices of the luminiferous medium are solenoidally aligned meaning that the individual rotations produce a similar effect to that of one single large rotation. A particle moving in the equatorial plane of these solenoidally aligned vortices will experience a compound centrifugal force, $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$, and be deflected sideways.

In a planetary orbit, the gravitational force of the planet entrains a pocket of the luminiferous medium along with it in its orbital motion. The entrained region and the planet move as one like an egg yolk and the rest of the egg. The smaller planet's entrained region moves through the larger planet's entrained region like a bubble. A centrifugal repulsion acts between the tiny vortices at the interface of the two entrained regions as they shear past each other.

In the case of the Coriolis force when it is applied to planetary orbits, the tiny molecular vortices of the luminiferous medium are precessing due to the radial gravitational field. In this respect they behave somewhat like compound turbines, in that where a turbine spins when exposed to a wind, a luminiferous molecular vortex is already spinning, but then precesses when it is subjected to an aether wind. The gravitational field constitutes a radial wind of pure stretched aether. When a planet moves only in the radial direction in a gravitational field, the effect of the precessing molecular vortices all around it is symmetrical, and so there will be no deflection. However, in situations where we have transverse motion occurring in addition to the radial motion, such as in non-circular orbits, an asymmetry will ensue resulting in a compound centrifugal force acting in the transverse direction, such as to cause the conservation of angular momentum.

In the case of rigid body rotations, a variation of the compound centrifugal force principle involves the spin of the actual molecules of the rigid body itself. In this respect it is convenient to consider the individual molecules of a gyroscope to be compound turbines. As the gyroscope rotates, its molecules are exposed to a circulating wind of luminiferous medium which causes them to precess with their precession axes parallel to the wind. The faster the motion the faster will be the precession and the molecules will become aligned such that their precession axes trace out concentric circles within the gyroscope. The situation is closely related to Ampère's circuital law and the solenoidal alignment of the precession axes is the basis of gyroscopic stability. The alignment induces a radial compound centrifugal pressure that has the effect of causing the gyroscope to resist external torque. In a magnetic field, the same reactive effect is known as inductance. There is no official recognition of the concept of spin-based inductance in the literature [8], yet when an external torque is applied to a spinning gyroscope it is quite clear that a strong reactance can be felt which cannot be accounted for by the moment of inertia. This reactance, which feels like magnetic repulsion, is a manifestation of Lenz's law extended to the gyroscope. It is the opposition which accompanies an induced effect.

The induced effect is the Coriolis force and it can be explained by considering a large gyroscope spinning clockwise with the cardinal points marked on the clock face in the inertial frame. If we force precess the gyroscope about a north-south axis in its plane of spin, there will be a change of the angle of attack of the 'electric wind' of the luminiferous medium that is circulating inside the gyroscope, and so the instantaneous effect will be for the precession axes of the individual molecules to become realigned within the gyroscope. The compound centrifugal force will hence act out of the gyroscope's plane of

rotation causing a large scale torque to act at right angles to the forced precession, while also restoring the solenoidal alignment of the molecules within the gyroscope. A forced precession on a north-south axis, east downwards, will therefore result in the gyroscope tilting upwards at the north point and downwards at the south point. A large scale Coriolis torque will have been induced about the east-west axis, caused by a small scale Coriolis force that is acting on the individual constituent molecules. Meanwhile the energy from the forced precession will be converted into simple spin in the new plane of rotation. Gyroscopic stability is a kind of sponge for spin.

This should not be seen as a case of Coriolis force being explained in terms of itself on a smaller scale, because on the smaller scale the circumstances are different and a distinct cause has been identified. On the smaller scale the molecules of the gyroscope are actually moving through the sea of even smaller molecular vortices that make up the luminiferous medium. It is this motion of a spinning molecule through the luminiferous medium that generates a differential centrifugal force on either side of the molecule, just like when a spinning cricket ball moves through the air.

In the case of the rattleback, the apparent simple centrifugal force on the large scale is being caused by a Coriolis force on the molecular scale.

Polar Coordinates in the Inertial Frame of Reference

VII. The mainstream literature contains two common methods for deriving the inertial forces. One method invokes the use of polar coordinates in an inertial frame of reference while the other method invokes a rotating frame of reference. Close examination of these two methods however exposes them to be identical. Let's first of all look at the derivation using polar coordinates without invoking a rotating frame of reference.

Consider a particle in motion in an inertial frame of reference. We write the position vector of this body relative to any arbitrarily chosen polar origin as,

$$\mathbf{r} = r\hat{\mathbf{r}} \tag{1}$$

where the unit vector $\hat{\mathbf{r}}$ is in the radial direction and where r is the radial distance. Taking the time derivative and using the product rule, we obtain the velocity term,

$$\dot{\mathbf{r}} = \dot{r}\hat{\mathbf{r}} + r\dot{\theta}\hat{\mathbf{\theta}} \tag{2}$$

where $\hat{\theta}$ is the unit vector in the transverse direction and where $\dot{\theta}$ is the angular speed about the polar origin. Taking the time derivative for a second time, we obtain the expression for acceleration in the inertial frame,

$$\ddot{\mathbf{r}} = \ddot{r}\hat{\mathbf{r}} + \dot{r}\dot{\theta}\hat{\mathbf{\theta}} + \dot{r}\dot{\theta}\hat{\mathbf{\theta}} + r\ddot{\theta}\hat{\mathbf{\theta}} - r\dot{\theta}^2\hat{\mathbf{r}}$$
(3)

which can be rearranged as,

$$\ddot{\mathbf{r}} = (\ddot{r} - r\dot{\theta}^2)\hat{\mathbf{r}} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\mathbf{\theta}}$$
(4)

The centrifugal force and the Coriolis force appear as the first and the third terms on the right hand side of equation (4). Note that no rotating frame of reference is needed, and that all that is necessary is to identify a centre of rotation. Contrary to popular belief, centrifugal force is a product of absolute rotation and not of circular motion. In the case of uniform straight line motion, the total acceleration will be zero, and hence we can deduce that the centrifugal force takes on the same mathematical form as the second (centripetal term) term on the right hand side of equation (4). It should also be noted that while the centrifugal force is specifically a radial force, the Coriolis force is specifically a transverse force. Uniform straight line motion corresponds to the inertial path, and so we can conclude that the inertial forces are the underlying cause of the inertial path, Newton's first law of motion, and the law of conservation of angular momentum.

Let's now take a look at the alternative derivation of the inertial forces which is prominent in the literature. It begins in the same way by considering the position vector of a particle, but this time the particle is specified to be undergoing circular motion. However, after establishing the velocity equation for the case of circular motion, the general case is then considered and the velocity equation is extended to,

$$(d\mathbf{r}/dt)_{S} = (\delta \mathbf{r}/\delta t)_{R} + \boldsymbol{\omega} \times \mathbf{r}$$
 (5)

where $(d\mathbf{r}/dt)_s$ is the velocity of the particle relative to the inertial frame and $\boldsymbol{\omega}$ is the angular velocity of the particle relative to the origin. Equation (5) corresponds exactly in every respect to equation (2), therefore the mention of circular motion and a rotating frame of reference at the beginning of the derivation is a red herring, which is very misleading.

It should further be noted in particular that the $(\delta \mathbf{r}/\delta t)_R$ term on the right hand side of equation (5), being the extended term not arising from the rotatory effect, does not therefore have any transverse component. This is an important fact which is never highlighted in the literature. This term is specifically the radial velocity term, a fact that by the use of unit vectors is evident in the polar

coordinates derivation above, and this radial term then shows up in the Coriolis force term. And since the Coriolis force term takes on the vector cross product format, $2\omega \times (\delta \mathbf{r}/\delta t)_R$, the Coriolis force must be a transverse force, exactly as is established in the polar coordinates derivation with respect to the inertial frame of reference.

The consequence of failing to specify that the Coriolis force is strictly a transverse force is that there prevails a misinformed belief that the Coriolis force can act in any direction. The consequence of confusing the situation by the introduction of a rotating frame of reference is that there prevails a further misinformed belief that the centrifugal force is merely a product of the rotation of the frame of reference itself rather than a product of the absolute rotation of the particle relative to the inertial frame. The latter error leads to the bizarre notion that a particle at rest, when observed from a rotating frame of reference, experiences a fictitious outward centrifugal force in the radial direction, even though circular motion can only induce transverse artefacts. This discrepancy is then patched up with an even more bizarre argument involving a radial Coriolis force. The argument runs that since the stationary object, as observed from the rotating frame of reference, is seen due to its inertia to trace out a circular path, there must exist a fictitious centripetal force acting upon it which can be justified as being the resultant of the outward radial fictitious centrifugal force and an inward radial fictitious Coriolis force.

While this is clearly wrong mathematically as well as being arrant nonsense in its own right, this is the argument which is nevertheless used in modern physics in order to mask the fact that both the centrifugal force and the Coriolis force are real forces. The centrifugal force, rather than being a real outward force that can pull a string taut or reverse the direction of a rotating rattleback, is reduced to a mere artefact of making observations from a rotating frame of reference. The active outward physical effect of centrifugal force, which is the very essence of the common understanding of the concept, is being denied by using a mathematical conjuring trick. Attributing the cause of the inertial forces to a rotating coordinate system is a fraudulent way of thinking which seems to be part and partial of the modern relativity culture where there are no absolutes, and it appears to be inspired with the intention of denying the existence of the absolute motion that is clearly exposed by Newton's bucket. This in turn seems to be aimed at denying the existence of the luminiferous medium.

In the parts of the mainstream literature where centrifugal force is derived from polar coordinates in the inertial frame without the involvement of a rotating frame of reference, the ensuing cognitive dissonance is typified by this quote from Marion [9], which was made in the context of the centrifugal term in the radial planetary orbit equation,

This quantity is traditionally called the *centrifugal force*, although it is not a "force" in the ordinary sense of the word. We shall, however, continue to use this unfortunate terminology since it is customary and convenient. Jerry B. Marion, 1965

So on the one hand they teach that centrifugal force is merely an artefact of making observations from a rotating frame of reference, yet in situations where no rotating frame of reference is involved and where the centrifugal force is real enough to hold a planet up or to snap a string, they choose to apologize for using the name 'centrifugal force', considering it to be an unfortunate terminology despite its convenience. Indeed more often they simply avoid using the term 'centrifugal force' altogether. The unpalatable truth however is that we can perceive and feel the outward effect that is induced by rotation, without the need to introduce a rotating coordinate system.

Conclusion and Discussion

VIII. It has been suggested by the late Professor Eric Laithwaite that there must exist an active spin-induced force and a spin-induced inductance, both of which remain unrecognized in classical mechanics. [8]

There is no doubt about it that when we apply a precessional torque to a spinning gyroscope we feel a very distinct reactance. It feels like magnetic repulsion and it is stronger than any reactance that could be accounted for by the moment of inertia. This reactance is the basis of gyroscopic stability. Another counterintuitive feature of spinning gyroscopes is that when they are placed on a pivot, they defy gravity in a manner which parallels the gravity defying effect that a horizontal magnetic field produces on a falling charged particle. Spin therefore induces real physical effects that must involve a mechanism similar in principle to that which lies behind electromagnetic phenomena.

The medium for the propagation of light (luminiferous medium), which is the cause of magnetic phenomena, can be shown to account for a spin-induced compound centrifugal force that has a similar mathematical structure to the Coriolis force. This would suggest that the spin-induced force has been hiding in plain view all along as a fictitious force that is merely an artefact of making observations from a rotating frame of reference, and this would appear to be one of the greatest deceptions in modern physics along with Einstein's theories of relativity.

It is proposed that contrary to what it says in the literature, the Coriolis force is a real non-intuitive force which obeys Newton's laws of motion, but which cannot be predicted by them. It is observable from an inertial frame of reference and often disguised in the inertial path or the law of conservation of angular momentum. Artefacts do arise in a rotating frame of reference, always

in the transverse direction, since we are merely imposing a circular motion on top of an already existing situation, but such transverse artefacts are not the thing that is being described by the Coriolis force formula. The Coriolis force formula specifically demands that a radial motion exists simultaneously with an independent but physically connected rotational motion. This requires that the radial motion be somehow constrained to its radial path during the rotation of the system. The literature generally confuses the situation by mixing up real Coriolis force problems with unconstrained scenarios such as that of a cannon firing a ball across a rotating platform, where no Coriolis force exists.

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"Long ago he (mankind) recognized that all perceptible matter comes from a primary substance, of a tenuity beyond conception and filling all space - the Akasha or luminiferous ether - which is acted upon by the lifegiving Prana or creative force, calling into existence, in never ending cycles, all things and phenomena. The primary substance, thrown into infinitesimal whirls of prodigious velocity, becomes gross matter; the force subsiding, the motion ceases and matter disappears, reverting to the primary substance". http://www.rastko.rs/istorija/tesla/oniell-tesla.html

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