Wikipedia and Coriolis Force

Frederick David Tombe,
sirius184@hotmail.com
14th September 2015

Abstract. **Wikipedia is the on-line encyclopaedia that anybody can edit. The content changes on a daily basis. One of the rules for editing is that editors must not insert original research. The contents must reflect exactly what is stated in reliable sources.**

Reliable sources present *Coriolis force* as an artefact of making observations from a rotating frame of reference. When stationary objects are viewed from a rotating frame of reference, it's true that the effects are unequivocally fictitious. However, in situations where an entire system is rotating, the Coriolis force can be very real, and this fact tends to cause confusion. It will be argued that in the former case scenarios there is no Coriolis force present at all, real or fictitious, while in the latter case scenarios the Coriolis force is due to Newton's laws of motion, and that it already exists independently of the rotating frame of reference.

Introduction

I. In the mainstream literature, a *Coriolis force* of the mathematical form $2m\mathbf{v}\times\boldsymbol{\omega}$ can apply to both a transverse inertial effect and also to a component that arises in the differential between two radial effects, each with the mathematical form $m\mathbf{v}\times\boldsymbol{\omega}$, in the context of a rotating frame of reference. The transverse inertial force is real, while the latter compound case scenarios may be real or fictitious depending on the circumstances. The literature however treats Coriolis force as one unified theory, and needless to say the editors at the Wikipedia article have been unable over the years to explain the topic in a singularly unambiguous fashion. We will now examine a few case scenarios that come under the umbrella term of Coriolis force in the mainstream literature.

The Apparent Circular Path Traced out by the Stars in the Sky

II. A popular scenario amongst Wikipedia editors is the fictitious circular path that is traced out by the stars in the sky due to the diurnal motion of the Earth. The argument runs that this fictitious circular path requires a fictitious centripetal force. It is then said that this fictitious centripetal force is the sum of an outward acting fictitious centrifugal force and an inward acting fictitious Coriolis force that is twice as large.

Let's consider the general case of an object undergoing circular motion about the centre of rotation of a rotating frame of reference, as observed from

within that frame. Let $\boldsymbol{n}$ be the fraction of the apparent angular speed of that object relative to the rotating frame, as compared to the angular speed of the rotating frame itself, relative to the inertial frame. When $\boldsymbol{n} = -1$ we have a special case corresponding to the situation where an object that is at rest in the inertial frame is viewed from the rotating frame, while $\boldsymbol{n} = 0$ corresponds to the co-rotating case scenario. If we take the magnitude of the centrifugal force connected with the rotating frame to be 1, then in general, the magnitude of the actual inward centripetal force that will be required to hold the object in circular motion will be $(1 + \boldsymbol{n})^2$. This centripetal force has to be actively applied. When we view the situation from the rotating frame, the apparent centripetal force will be different, this time having a magnitude of $\boldsymbol{n}^2$, and the difference between the real and the apparent centripetal forces will therefore be $1 + 2\boldsymbol{n}$. It is claimed that this discrepancy of $1 + 2\boldsymbol{n}$ is accounted for by the sum of the centrifugal force and a radial Coriolis force, the latter being twice as large by virtue of having a factor of 2 in its formula, $2m\mathbf{v}\times\boldsymbol{\omega}$.

It is of particular significance that these purely fictitious effects are derived from a compound of two radial effects, one real and one fictitious, each with the mathematical form $m\mathbf{v}\times\boldsymbol{\omega}$. The significance of this fact will now be explored further.

The Eötvös Effect

III. The idea of *Coriolis force* first originated in a paper written in 1835 by French scientist Gaspard Gustave de Coriolis in connection with water wheels [1]. A feature of note in G.G. Coriolis's paper is his use of the term *compound centrifugal force* for the force that would later bear his name. Interestingly, while the name *compound centrifugal force* would suggest that a Coriolis force arises as a compound of two centrifugal forces, as would the formula $2m\mathbf{v}\times\boldsymbol{\omega}$ with its factor of 2, there is no clear evidence that G.G. Coriolis himself saw it this way. Nevertheless, the Coriolis force formula can indeed be established as a differential between two inertial centrifugal forces. This is seen in the case of east-west air currents in the atmosphere where an apparent force in the form of a Coriolis force appears in the Earth's rotating frame. This is a real effect and its vertical component is known as the *Eötvös Effect*. The Coriolis force format follows from the expansion of the square of a sum into a polynomial. For example, if v is the circumferential speed of the Earth's surface due to its rotation, and u is the speed of an east-west air current relative to the Earth's surface, then the difference between the centrifugal force acting on air that is stationary relative to the Earth and the centrifugal force acting on air that is moving at speed u relative to the Earth, will be $(v + u)^2 - v^2$, which is equal to $u^2 + 2uv$. The u^2 term is insignificant in magnitude.

This is the same mathematical pattern as in section **II** above. The end result is derived from the compound of two radial effects each of the form m**v**×**ω**, and in neither case does the centrifugal-like term in the end result actually refer to a real centrifugal force as such.

An actual inertial centrifugal force on the other hand follows from Newton's first law of motion and it can cause a Newton's third law reaction when the object acts on a constraint. The Wikipedia editors are prepared to recognize the existence of this Newton's third law effect, albeit somewhat reluctantly. They even recognize that it is real and frame independent, but they adamantly refuse to recognize the primary Newton's first law effect that causes it. They'll accept its reality if it is described as being an effect of inertia, but under no circumstances will they acknowledge that this effect of inertia is an inertial centrifugal force. And because of this reticence to acknowledge the primary cause, they get cause and effect reversed and they consider the action of the object on the constraint, due to the object's inertia, to be the reaction. And even then they palm off this real effect into a separate article under the misnomer of *"Reactive centrifugal force"* as if it's a separate topic altogether, nothing to do with inertial centrifugal force. Meanwhile the primary inertial centrifugal force itself is also palmed off, even more discreetly, into another article entitled *"Polar coordinate system"* without so much as a link to it in the main centrifugal force article. The final part of the deception is then to equate the term *fictitious force* with *inertial force* and to present the fictitious effects described in section **II** above as being the flagship examples of the inertial forces. There is a determination to make readers believe that the inertial forces are not real, and this determination knows no bounds.

As regards the Coriolis-like terms, in the fictitious case scenario described in section **II**, this will act inwards in retrograde case scenarios, yet if the constraint is removed, the object will fly off tangentially, and hence outwards. It does not therefore equate with the very real inertial Coriolis force that is observed in cyclones. The Eötvös Effect on the other hand, along with its horizontal counterpart, is a real effect because it is compounded from the difference between two real inertial centrifugal forces. In this case, the Coriolis-like term always acts radially outwards, except in a narrow band where the horizontal motion is both retrograde and smaller in magnitude as compared to the rotation of the Earth, where it then acts radially inwards. In the Earth's rotating frame it looks and acts just like a Coriolis force, but it is the difference between two inertial forces rather than being an inertial force in its own right. It is not the compound centrifugal force that is derived in G.G. Coriolis's 1835 paper, but there must be a reason for the commonality. The original Coriolis force is a purely transverse inertial force that will be discussed in the next section along with the issue of its commonality with the Eötvös Effect.

Newton's First law of Motion

IV. It is often said that inertia is not a force, and that the uniform straight line inertial path of Newton's first law of motion is not associated with a force, and that neither does it produce a force. Ignoring the forces that arise in a collision, it is true that an object undergoing a uniform straight line inertial path does not have an associated linear force. It does however have associated radial and transverse forces from the perspective of any arbitrarily chosen polar origin.

When Newton's first law of motion is expressed in polar coordinates, relative to any arbitrarily chosen polar origin, radial and transverse inertial forces are revealed. See the derivation in the appendix in section **VII** at the end. The transverse force in particular is not overtly apparent when an object is undergoing its straight line inertial path, but it can be exposed in a rotating system. A radial motion in a rotating system will experience a transverse force which exactly corresponds to the mathematical term that is known as the Coriolis force. Dragging forces cause the co-rotation of the elements in the system, but if these dragging forces fail to constrain this Coriolis force, a transverse deflection will become apparent relative to a frame of reference that is fixed in the rotating system, and this deflection can cause a physical interaction with other elements in the rotating system.

The inertial Coriolis force as observed in a rotating system is simply the exposure of a Coriolis force that was already present in the straight line inertial path. It is not a fictitious effect. It is a real effect that has been isolated and contrasted against a rotating background. It is a consequence of rather than a supplement to Newton's laws. The Coriolis force interacts with other physical forces, something that fictitious forces can't do, and the combined effects can be observed from outside the rotating frame. An example of this is the cyclonic behaviour in the atmosphere which can be observed from outer space. A rotating system that is induced within a rotating system cannot be an artefact.

We know from the manner of the derivation that this transverse inertial Coriolis force is the original *compound centrifugal force* of G.G. Coriolis's 1835 paper, and as demonstrated in section **VII** of "Wikipedia and Centrifugal Force" **[2]**, repeated in the appendix at the end of this article, it can never be in the radial direction. The Eötvös Effect is radial because it is overtly a *compound centrifugal force*.

Newton's Third Law of Motion

V. The reality of the Coriolis force is further borne out by case scenarios involving Newton's third Law of Motion. In the case scenario in section **IV** above, if the transverse deflection in the rotating system is prohibited by a

constraint, for example in the case of a marble that is rolling along a radial groove in a rotating platform, the Coriolis force will instead cause the entire platform to either angularly accelerate or angularly decelerate. This is an example of Newton's third law of motion and the conservation of angular momentum, clearly exposing the Coriolis force to be a real force. The gyroscope provides another demonstration of the fact that the Coriolis force is a real force and not merely an artefact of making an observation from a rotating frame of reference. During a forced precession, the rigid structure of the gyroscope acts as a constraint which keeps the rim on a radial path with respect to the precession axis. Equal and opposite Coriolis forces on either side of the rim then cause the gyroscope to tilt at right angles to the forced precession. This is a very real effect which can even defy gravity when the gyroscope is placed on a pivot.

Conclusion

VI. Wikipedia's rule regarding *no original research* should not be used to sabotage the understanding of this controversial topic. The talk pages may well be intended for discussing improvements to the article, but improvements only come with better understanding. Better understanding will lead to better application of reliable sources.

The difference between two radial inertial centrifugal forces can lead to a mathematical form that looks like the sum of a radial centrifugal force and a radial Coriolis force. This happens with east-west air currents in the atmosphere and with the related Eötvös Effect. The fact that a Coriolis-like term can be derived as the mathematical combination of two centrifugal forces is significant. Maxwell believed that space is densely packed with tiny vortices that are pressing against each other with centrifugal force while striving to dilate. In 1861 when explaining the magnetic force on a current carrying wire, Maxwell said *" - - so that the vortices on the east side of the current will be more powerful than those on the west side"* [3]. In other words, the tiny vortices cause a differential centrifugal pressure to act on either side of the electric current. The magnetic force is therefore a *compound centrifugal force*. Maxwell derived this force to be $\mathbf{F} = \mu\mathbf{v}\times\mathbf{H}$, where $\mathbf{H}$ is the vorticity of the vortices, $\mathbf{v}$ is the electric current, and μ is related to the density of the vortices. Since $\mathbf{H} = 2\boldsymbol{\omega}$, where $\boldsymbol{\omega}$ is the angular velocity, the Coriolis force format $\mathbf{F} = 2\mu\mathbf{v}\times\boldsymbol{\omega}$ emerges.

The problem with the mainstream literature is that no distinction is made as between inertial effects that arise as a consequence of Newton's laws of motion on the one hand, and fictitious effects that arise from making observations in an accelerating frame of reference on the other hand. Fictitious effects supplement Newton's laws, while inertial effects are a consequence of Newton's laws. The Coriolis force arises relative to a polar origin, in connection with an object that

is following its inertial path. It is exposed in a rotating system through the transverse deflection of a radial motion, but it already exists independently of the rotating frame. An example is the deflection caused on north-south air currents in the atmosphere due to the Earth's rotation. On the other hand, the Coriolis-like radial effect that acts on east-west air currents in the atmosphere is not a Coriolis force. It is real, but it arises from the difference between two inertial centrifugal forces. The similarity of form however cannot just be a meaningless coincidence. It must be telling us that the inertial Coriolis force, just like the magnetic force which shares an identical mathematical form, is also a *compound centrifugal force* arising from the same deeper physical cause in Maxwell's sea of tiny vortices. G.G. Coriolis himself referred to the Coriolis force as a *compound centrifugal force* for reasons that are not altogether clear, but it seems in retrospect to have been a most appropriate name.

The inertial forces can prevent the planets from falling into the Sun, they can prevent a pivoted gyroscope from toppling under the force of gravity, they can make a rotating platform spin faster, and they can reverse the direction of a rotating rattleback. The Coriolis force in the atmosphere causes cyclonic effects which can be observed from outer space. None of these examples are fictitious effects. The inertial forces are not artefacts, and neither is a rotating frame of reference needed in order to observe any of these phenomena.

*(At the time of writing, although this could change at any moment, the Wikipedia article is wrongly named **Coriolis Effect**, which is a term used by the UK Met Office for a consequence of the Coriolis force in the atmosphere, but which is not the correct and general term for the inertial force itself.)*

Appendix - Polar Coordinates in the Inertial Frame of Reference

VII. Consider a particle in motion in an inertial frame of reference. We write its position vector relative to any arbitrarily chosen polar origin as,

$$\mathbf{r} = r\hat{\mathbf{r}} \qquad \textbf{(1)}$$

where the unit vector $\hat{\mathbf{r}}$ is in the radial direction and where r is the radial distance. Taking the time derivative and using the product rule, we obtain the velocity term,

$$\dot{\mathbf{r}} = \dot{r}\hat{\mathbf{r}} + r\dot{\theta}\hat{\boldsymbol{\theta}} \qquad \textbf{(2)}$$

where $\hat{\boldsymbol{\theta}}$ is the unit vector in the transverse direction and where $\dot{\theta}$ is the angular speed about the polar origin. Taking the time derivative for a second time, we obtain the expression for acceleration in the inertial frame,

$$\ddot{\mathbf{r}} = \ddot{r}\hat{\mathbf{r}} + \dot{r}\dot{\theta}\hat{\boldsymbol{\theta}} + \dot{r}\dot{\theta}\hat{\boldsymbol{\theta}} + r\ddot{\theta}\hat{\boldsymbol{\theta}} - r\dot{\theta}^2\hat{\mathbf{r}} \qquad \textbf{(3)}$$

which can be rearranged as,

$$\ddot{\mathbf{r}} = (\ddot{r} - r\dot{\theta}^2)\hat{\mathbf{r}} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\boldsymbol{\theta}} \qquad \textbf{(4)}$$

The first and the third terms on the right hand side of equation **(4)** are the centrifugal and the Coriolis terms respectively. Note that no rotating frame of reference is needed, and that all that is necessary is to identify a centre of rotation. Contrary to popular belief, centrifugal force is a product of absolute rotation and not of making observations from a rotating frame of reference. In the case of uniform straight line motion, the total acceleration will be zero, and hence we can deduce that the centrifugal force takes on the same mathematical form as the second (centripetal term) term on the right hand side of equation **(4)**. It should also be noted that while the centrifugal force is specifically a radial force, the Coriolis force is specifically a transverse force.

In the Wikipedia article on polar coordinates, *at the moment as I write this*, it says that the centrifugal and Coriolis terms above are lookalikes which are a mathematical consequence of differentiation, and hence they are not the real thing? Let's therefore take a look at the alternative derivation of the inertial forces which is prominent in the literature and which is eagerly supported by Wikipedia editors as being the only true way. This time the position vector **r** is tied up with a rotating frame of reference. The equation for a particle moving in the rotating frame is then written as,

$$(d\mathbf{r}/dt)_S = (\delta\mathbf{r}/\delta t)_R + \boldsymbol{\omega}\times\mathbf{r} \qquad \textbf{(5)}$$

where $(d\mathbf{r}/dt)_S$ is the velocity of the particle relative to the inertial frame, and **ω** is the angular velocity of the rotating frame. It is assumed that the velocity of the particle in the rotating frame, $(\delta\mathbf{r}/\delta t)_R$, can be in any direction, but if that is so, then **r** cannot be the same vector throughout the equation, since the origin of the latter will have to be the fixed point in the rotating frame which has the transverse speed **ω**×**r**. It's a simple question of vector addition of velocities, and so a serious error has been made. Equation **(5)** can only make sense if **r** is the same vector throughout the equation, but in that case it becomes equivalent in every respect to equation **(2)**, and therefore the meaning changes and the rotating frame of reference at the beginning of the derivation becomes irrelevant and misleading. The $(\delta\mathbf{r}/\delta t)_R$ term therefore cannot have any transverse component, and since the Coriolis force term takes on the vector cross product format, $2\boldsymbol{\omega}\times(\delta\mathbf{r}/\delta t)_R$, the Coriolis force must be strictly a transverse force. The consequence of this mathematics error is the absurd belief that the Coriolis force can act in any direction.

References

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